

Algorithms for NLP



Parsing I

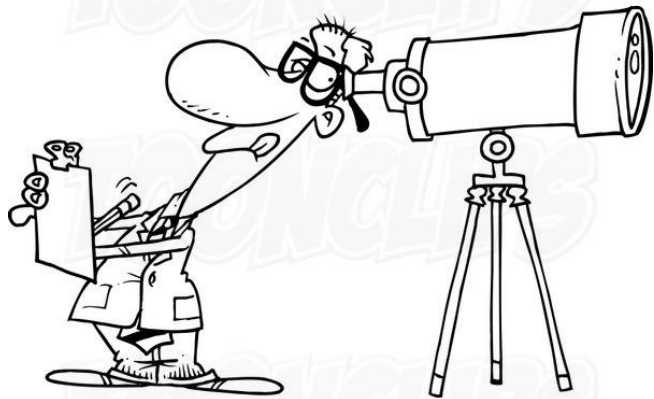
Yulia Tsvetkov – CMU

Slides: Ivan Titov – University of Edinburgh,
Taylor Berg-Kirkpatrick – CMU/UCSD, Dan Klein – UC Berkeley



Ambiguity

- I saw a girl with a telescope



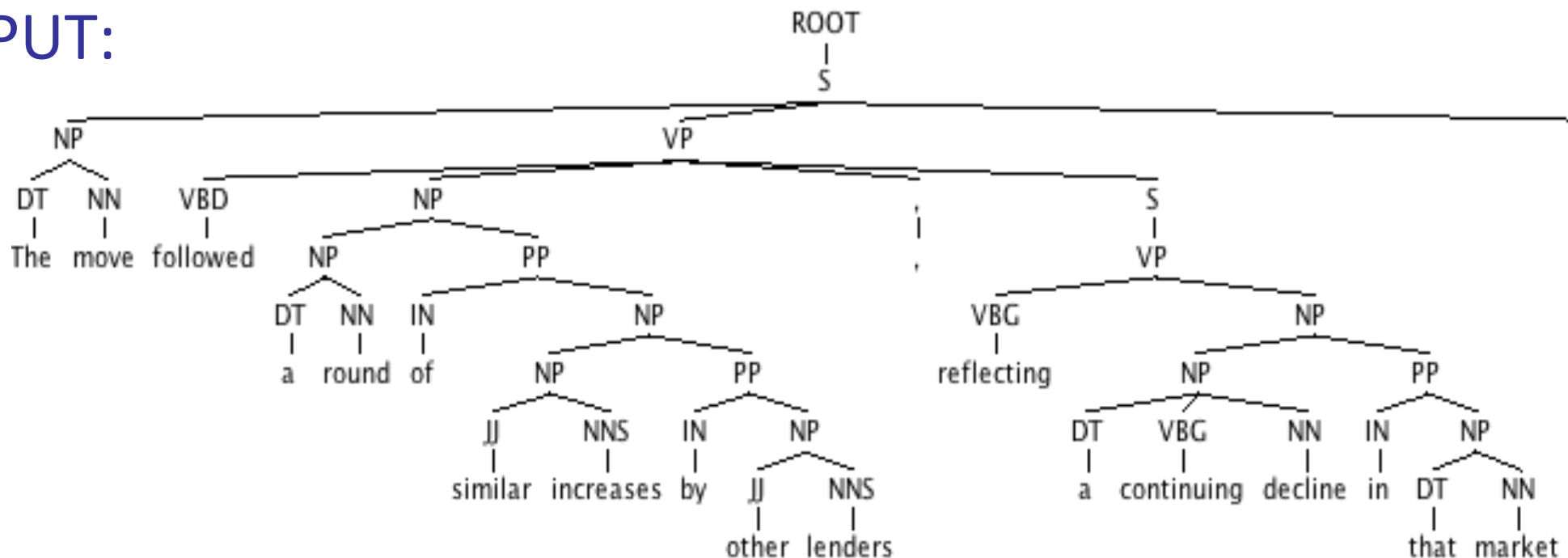


Parsing

- INPUT:

- The move followed a round of similar increases by other lenders, reflecting a continuing decline in that market

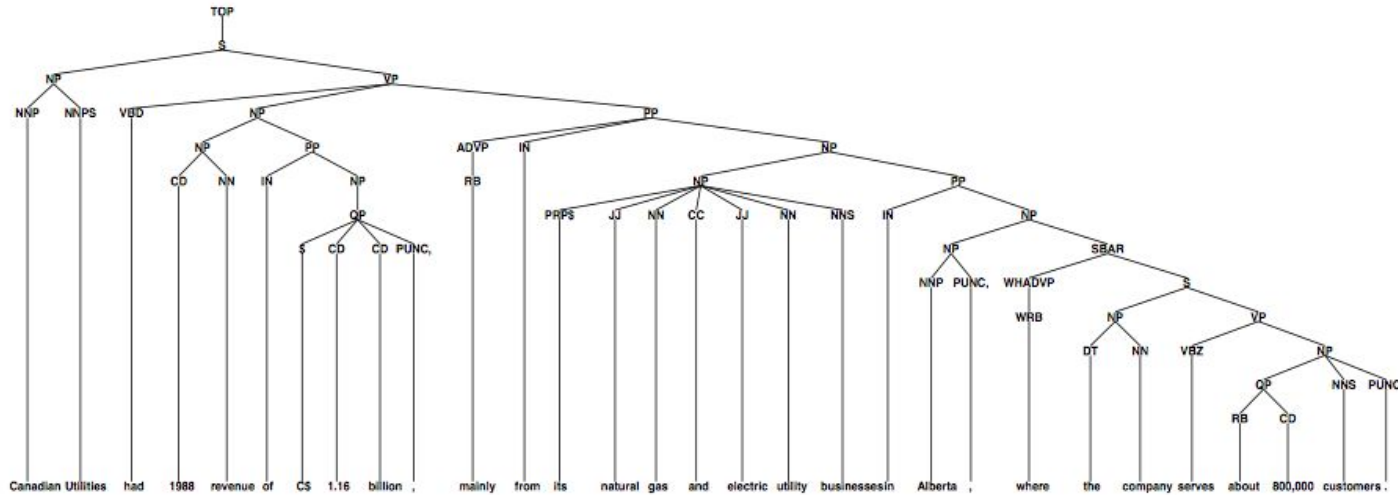
- OUTPUT:





A Supervised ML Problem

- Data for parsing experiments:
 - Penn WSJ Treebank = 50,000 sentences with associated trees
 - Usual set-up: 40,000 training, 2,400 test



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers [from Michael Collins slides]



Outline

- Syntax: intro, CFGs, PCFGs
- CFGs: Parsing
- PCFGs: Parsing
- Parsing evaluation

Syntax



Syntax

- The study of the patterns of formation of sentences and phrases from word
 - my dog Pron N
 - the dog Det N
 - the cat Det N

 - the large cat Det Adj N
 - the black cat Det Adj N

 - ate a sausage V Det N



Syntax

- The study of the patterns of formation of sentences and phrases from word
 - Borders with **semantics** and **morphology** sometimes blurred

Afyonkarahisarlılaştırabildiklerimizdenmişsinizcesine

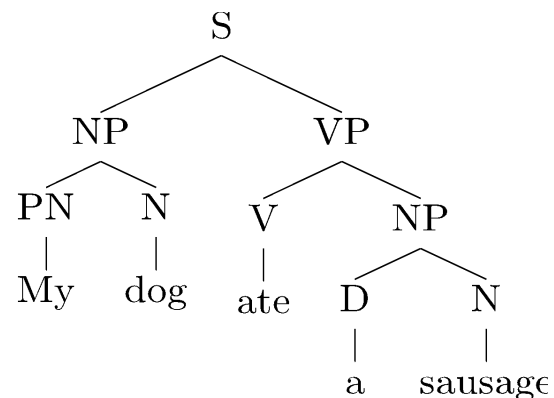
in Turkish means

"as if you are one of the people that we thought to be originating from Afyonkarahisar" [[wikipedia](#)]



Parsing

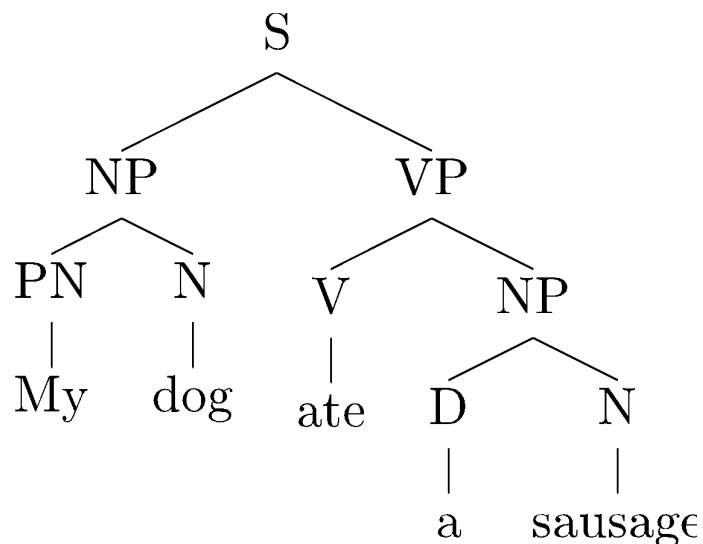
- The process of predicting syntactic representations
- Syntactic Representations
 - Different types of syntactic representations are possible, for example:



Constituent (a.k.a. phrase-structure) tree



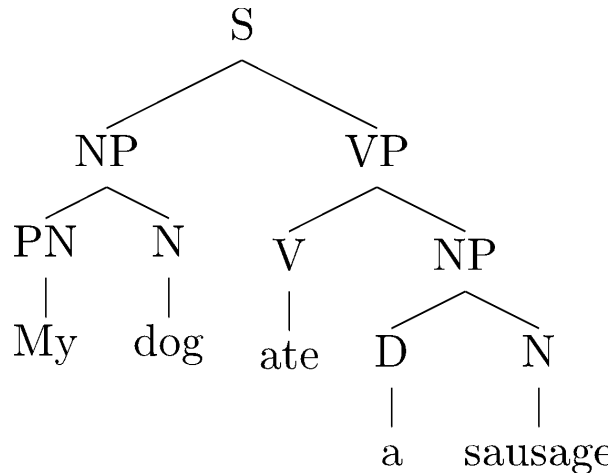
Constituent trees



- Internal nodes correspond to phrases
 - S – a sentence
 - NP (Noun Phrase): My dog, a sandwich, lakes, ..
 - VP (Verb Phrase): ate a sausage, barked, ...
 - PP (Prepositional phrases): with a friend, in a car, ...
- Nodes immediately above words are PoS tags (aka preterminals)
 - PN – pronoun
 - D – determiner
 - V – verb
 - N – noun
 - P – preposition



Bracketing notation



- It is often convenient to represent a tree as a bracketed sequence

(S

(NP (PN My) (N Dog))

(VP (V ate)

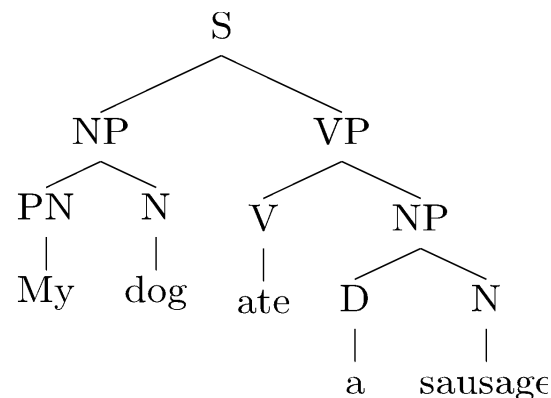
(NP (D a) (N sausage))

)

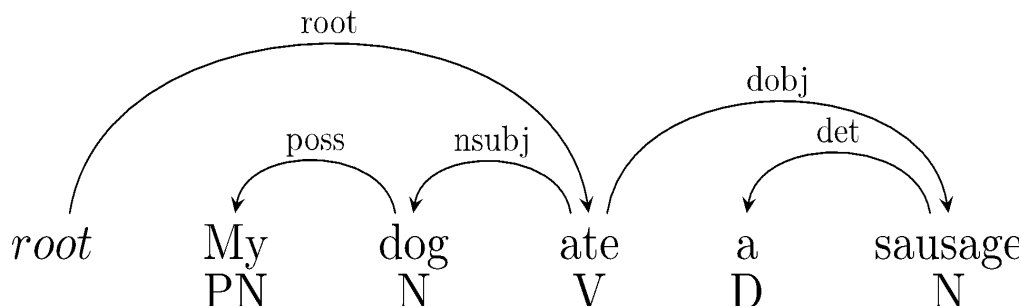


Parsing

- The process of predicting syntactic representations
- Syntactic Representations
 - Different types of syntactic representations are possible, for example:



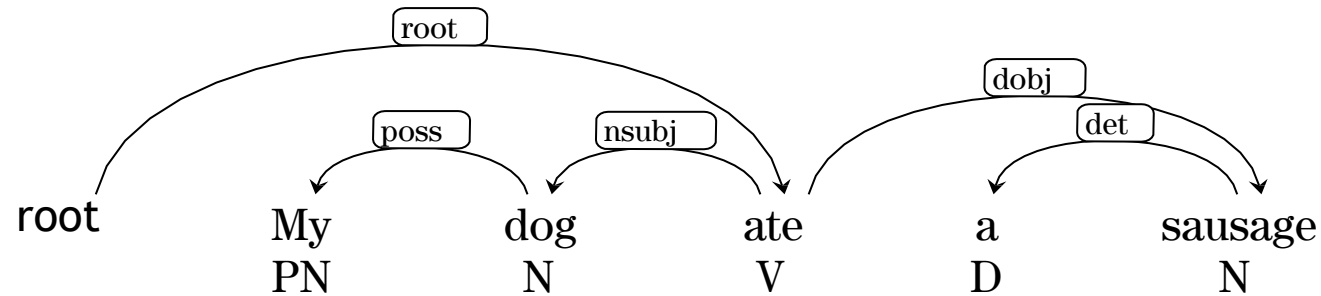
Constituent (a.k.a. phrase-structure) tree



Dependency tree



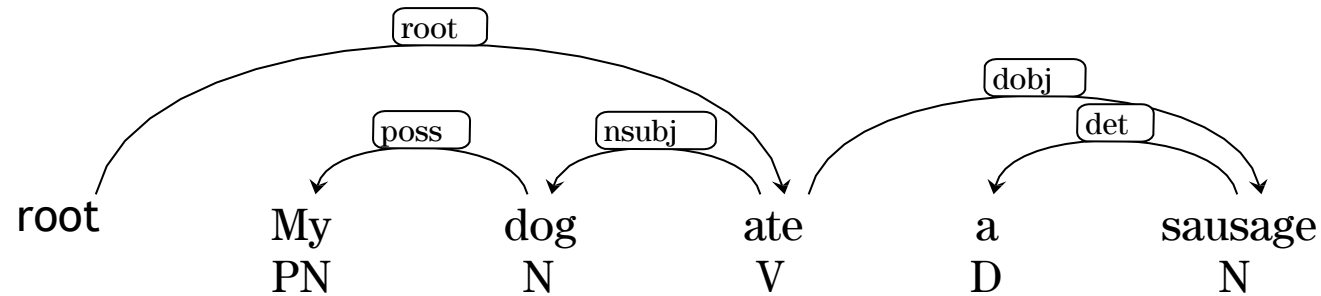
Dependency trees



- Nodes are words (along with PoS tags)
- Directed arcs encode syntactic dependencies between them
- Labels are types of relations between the words
 - poss – possessive
 - dobj – direct object
 - nsubj - subject
 - det - determiner



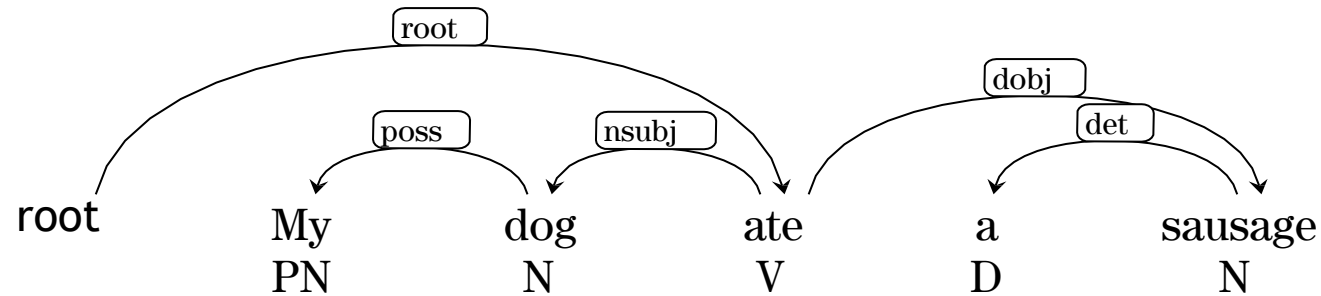
Recovering shallow semantics



- Some semantic information can be (approximately) derived from syntactic information
 - Subjects (nsubj) are (often) agents ("initiator / doers for an action")
 - Direct objects (dobj) are (often) patients ("affected entities")



Recovering shallow semantics

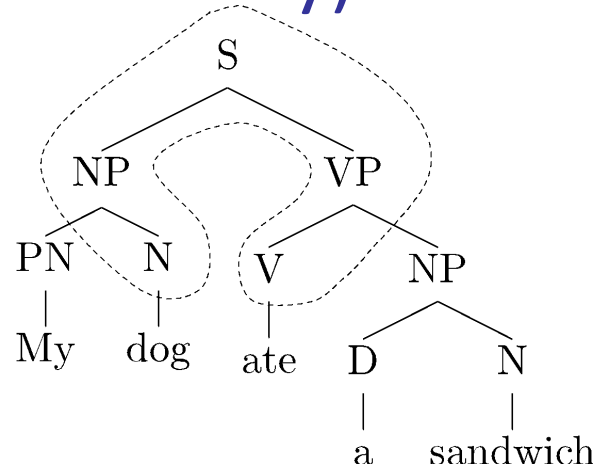


- Some semantic information can be (approximately) derived from syntactic information
 - Subjects (nsubj) are (often) agents ("initiator / doers for an action")
 - Direct objects (dobj) are (often) patients ("affected entities")
- But even for agents and patients consider:
 - Mary is baking a cake in the oven
 - A cake is baking in the oven
- In general it is not trivial even for the most shallow forms of semantics
 - E.g., consider prepositions: *in* can encode direction, position, temporal information, ...

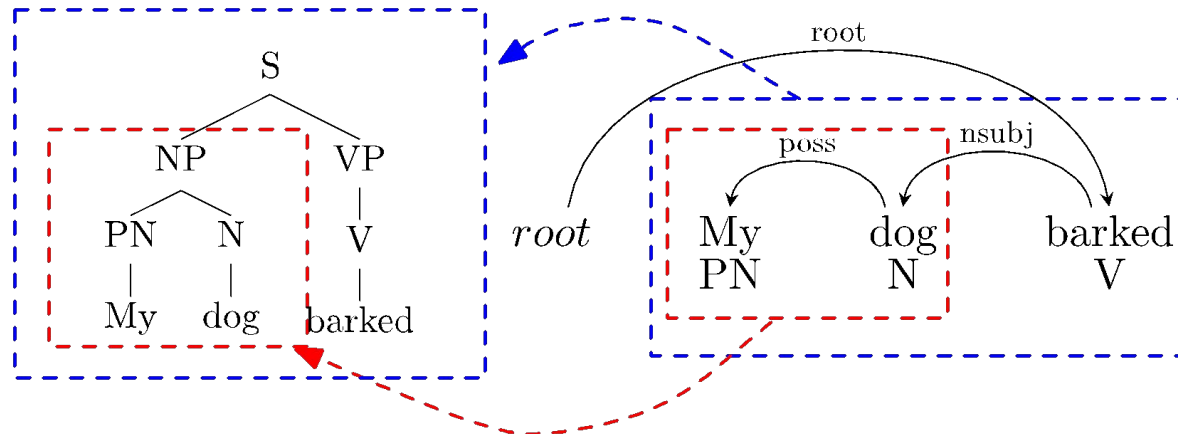


Constituent and dependency representations

- Constituent trees can (potentially) be converted to dependency trees

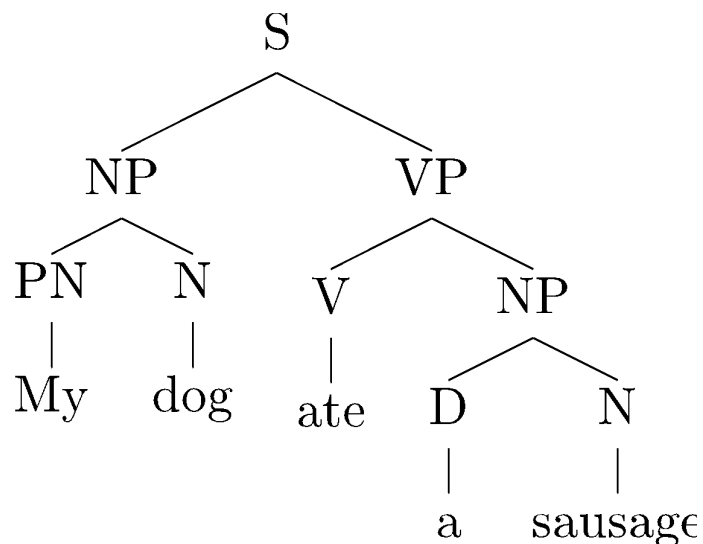


- Dependency trees can (potentially) be converted to constituent trees





Constituent trees



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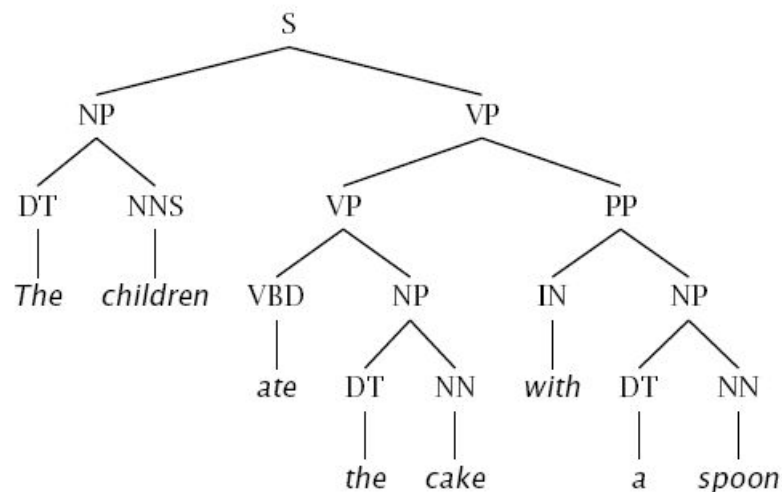


Constituency Tests

- How do we know what nodes go in the tree?

- Classic constituency tests:

- Substitution by *proform*
- Movement
 - Clefting
 - Preposing
 - Passive
- Modification
- Coordination/Conjunction
- Ellipsis/Deletion





Conflicting Tests

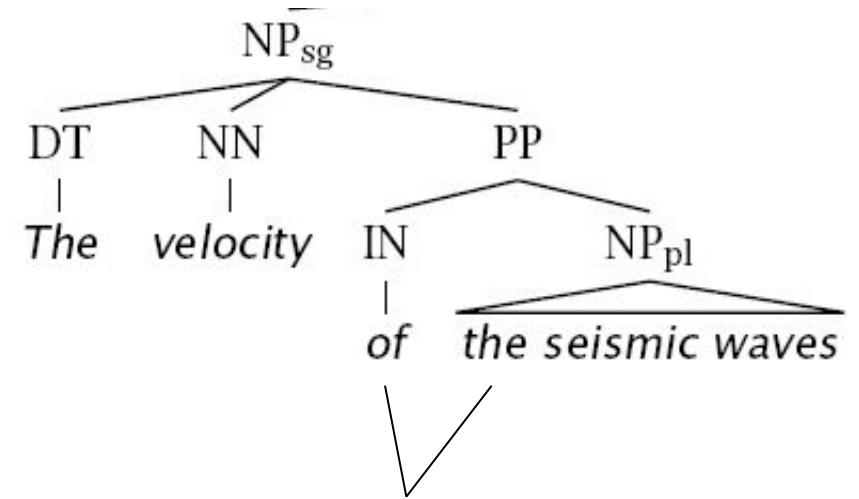
- Constituency isn't always clear

- Units of transfer:

- think about ~ penser à
 - talk about ~ hablar de

- Phonological reduction:

- I will go → I'll go
 - I want to go → I wanna go
 - a le centre → au centre



La vitesse des ondes sismiques

CFGs



Context Free Grammar (CFG)

Grammar (CFG)

ROOT \rightarrow S

S \rightarrow NP VP

NP \rightarrow DT NN

NP \rightarrow NN NNS

NP \rightarrow NP PP

VP \rightarrow VBP NP

VP \rightarrow VBP NP PP

PP \rightarrow IN NP

Lexicon

NN \rightarrow interest

NNS \rightarrow raises

VBP \rightarrow interest

VBZ \rightarrow raises

...

- Other grammar formalisms: LFG, HPSG, TAG, CCG...



Treebank Sentences

```
( (S (NP-SBJ The move)
  (VP followed
    (NP (NP a round)
      (PP of
        (NP (NP similar increases)
          (PP by
            (NP other lenders))
          (PP against
            (NP Arizona real estate loans))))))
    ,
    (S-ADV (NP-SBJ *)
      (VP reflecting
        (NP (NP a continuing decline)
          (PP-LOC in
            (NP that market))))))
  .))
```



CFGs

S

$S \rightarrow NP VP$

$VP \rightarrow V$

$VP \rightarrow V NP$

$VP \rightarrow VP PP$

$NP \rightarrow NP PP$

$NP \rightarrow D N$

$NP \rightarrow PN$

$PP \rightarrow P NP$

$N \rightarrow girl$

$N \rightarrow telescope$

$N \rightarrow sandwich$

$PN \rightarrow I$

$V \rightarrow saw$

$V \rightarrow ate$

$P \rightarrow with$

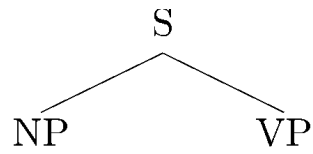
$P \rightarrow in$

$D \rightarrow a$

$D \rightarrow the$



CFGs



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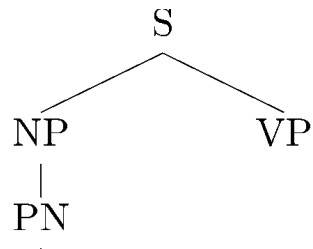
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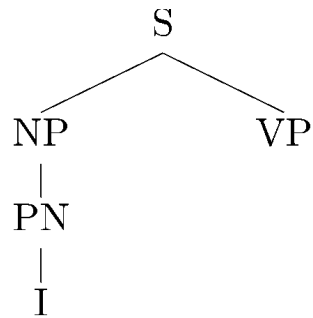
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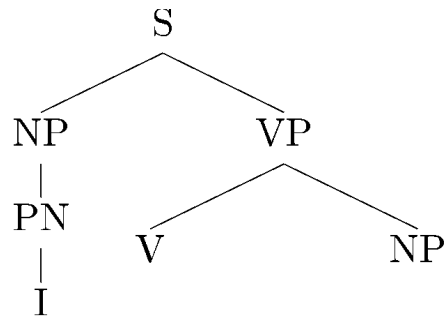
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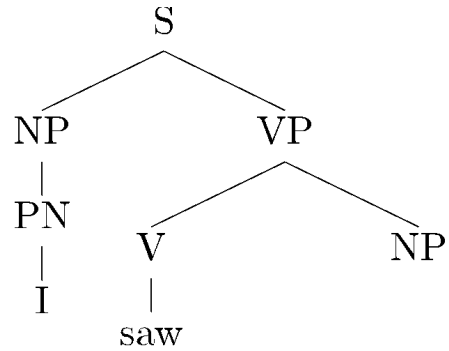
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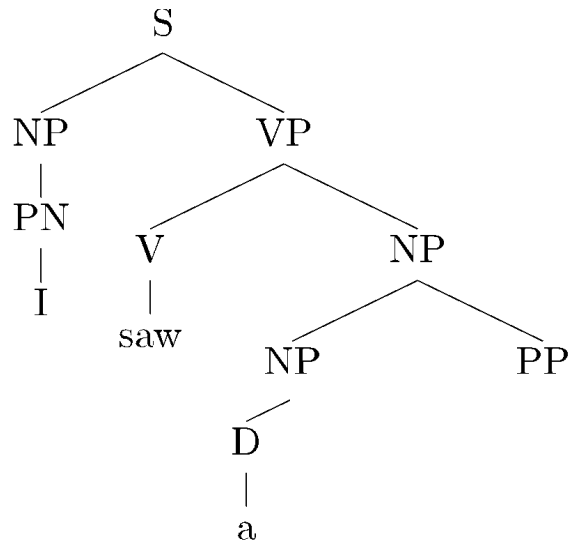
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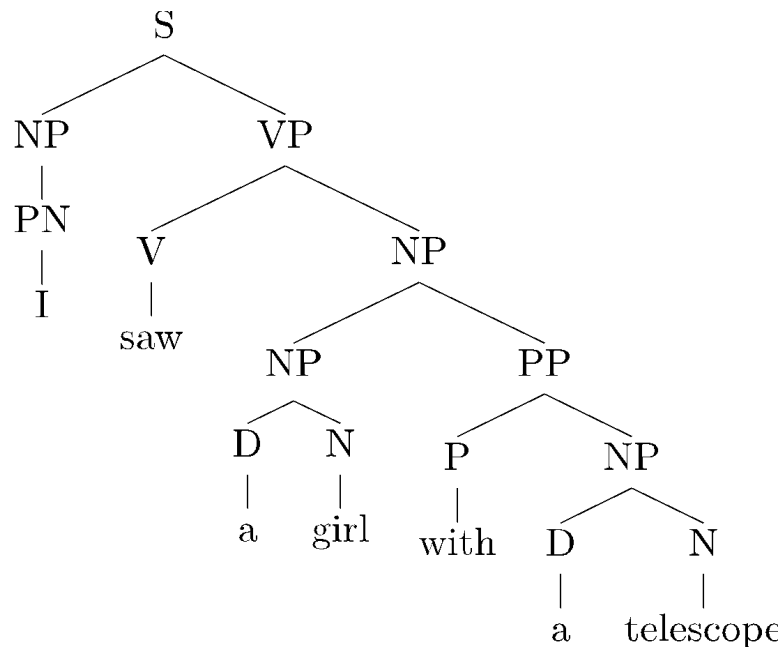
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Context-Free Grammars

- A context-free grammar is a 4-tuple $\langle N, T, S, R \rangle$
 - N : the set of non-terminals
 - Phrasal categories: S, NP, VP, ADJP, etc.
 - Parts-of-speech (pre-terminals): NN, JJ, DT, VB
 - T : the set of terminals (the words)
 - S : the start symbol
 - Often written as ROOT or TOP
 - *Not* usually the sentence non-terminal S
 - R : the set of rules
 - Of the form $X \rightarrow Y_1 Y_2 \dots Y_k$, with $X, Y_i \in N$
 - Examples: $S \rightarrow NP VP$, $VP \rightarrow VP CC VP$
 - Also called rewrites, productions, or local trees



An example grammar

$N = \{S, VP, NP, PP, N, V, PN, P\}$

$T = \{girl, telescope, sandwich, I, saw, ate, with, in, a, the\}$

$S = \{S\}$

$R :$

Called **Inner rules**

$S \rightarrow NP VP$ (NP A girl) (VP ate a sandwich)

$VP \rightarrow V$

$VP \rightarrow V NP$ (V ate) (NP a sandwich)

$VP \rightarrow VP PP$ (VP saw a girl) (PP with a telescope)

$NP \rightarrow NP PP$ (NP a girl) (PP with a sandwich)

$NP \rightarrow D N$ (D a) (N sandwich)

$NP \rightarrow PN$

$PP \rightarrow P NP$ (P with) (NP with a sandwich)

Preterminal rules

$N \rightarrow girl$

$N \rightarrow telescope$

$N \rightarrow sandwich$

$PN \rightarrow I$

$V \rightarrow saw$

$V \rightarrow ate$

$P \rightarrow with$

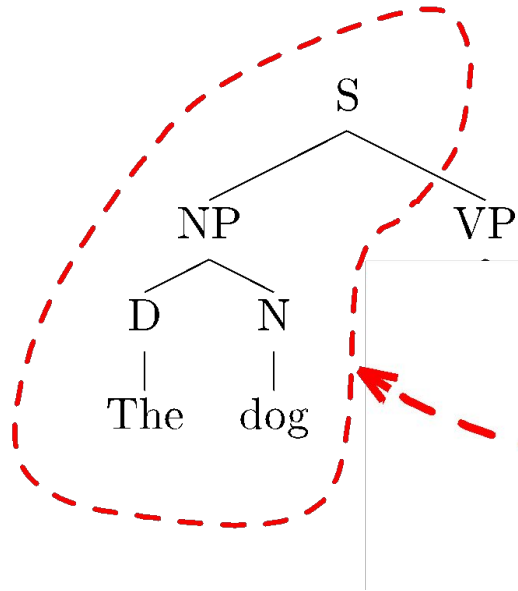
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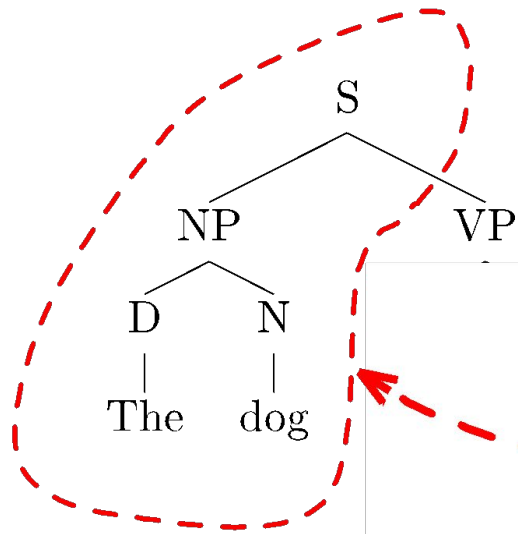
Why context-free?



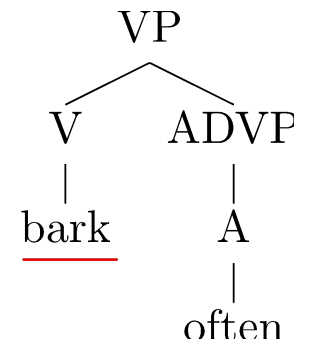
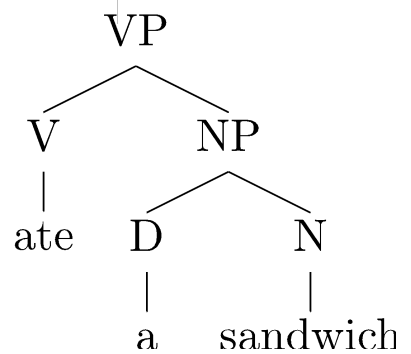
What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**



Why context-free?



What can be a sub-tree is only affected by what the phrase type is (VP) but not the **context**

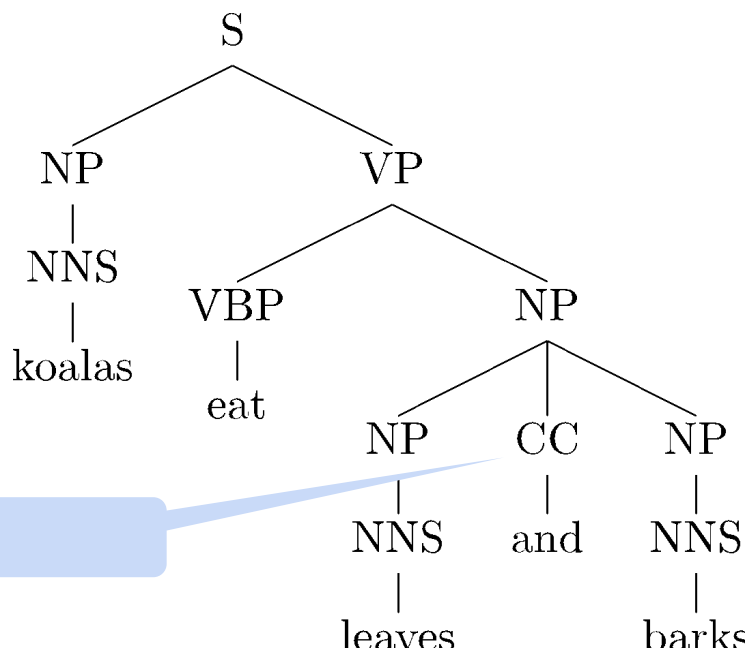


Not grammatical



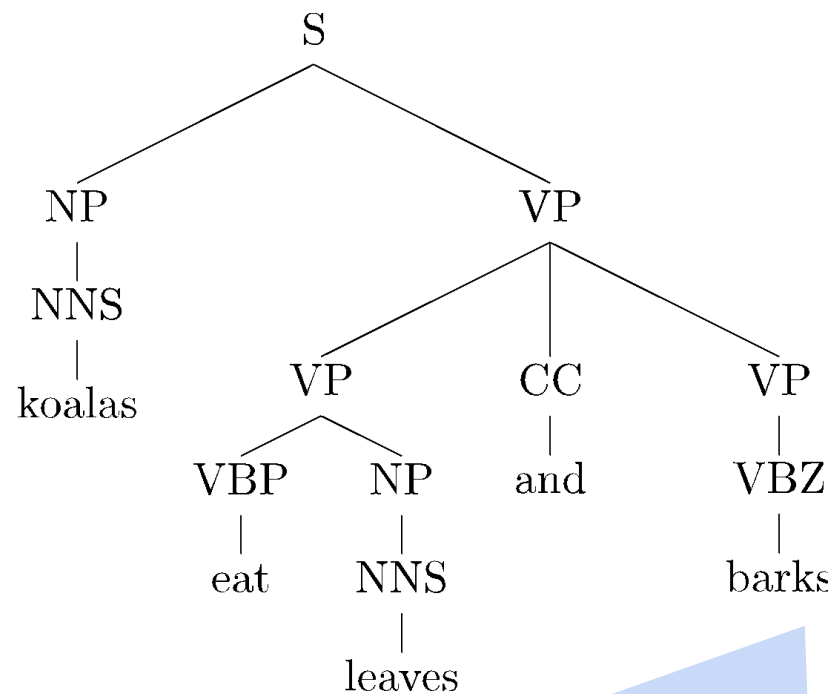
Coordination ambiguity

- Here, the coarse VP and NP categories cannot enforce subject-verb agreement in number resulting in the coordination ambiguity



Coordination

"Bark" can refer both to a noun or a verb



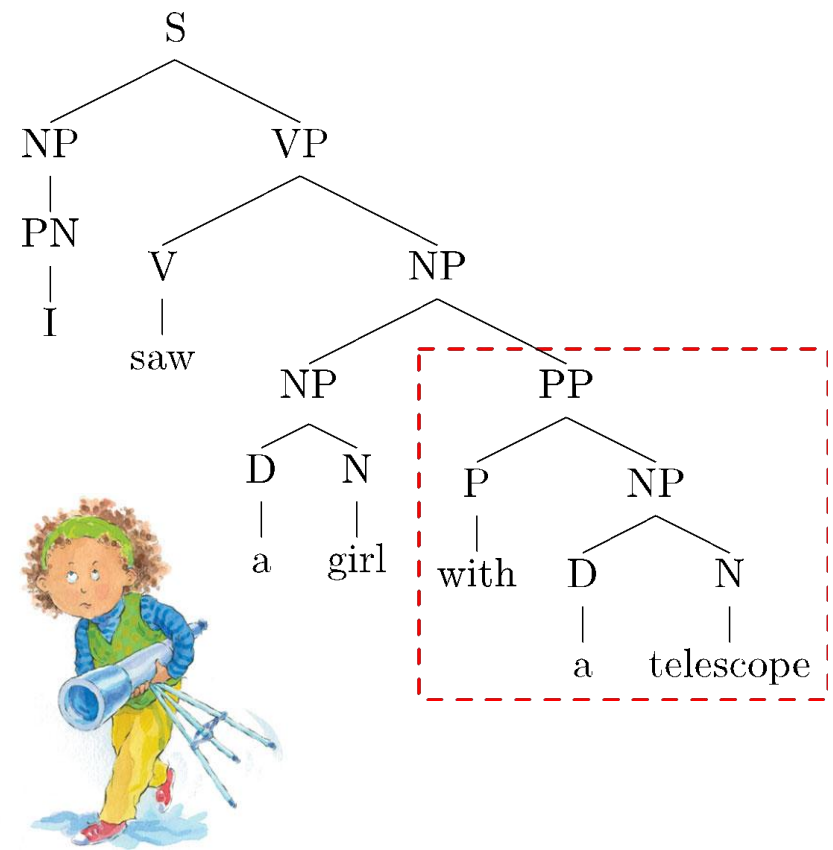
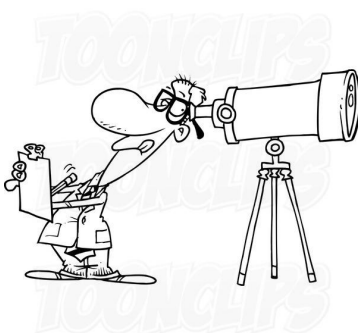
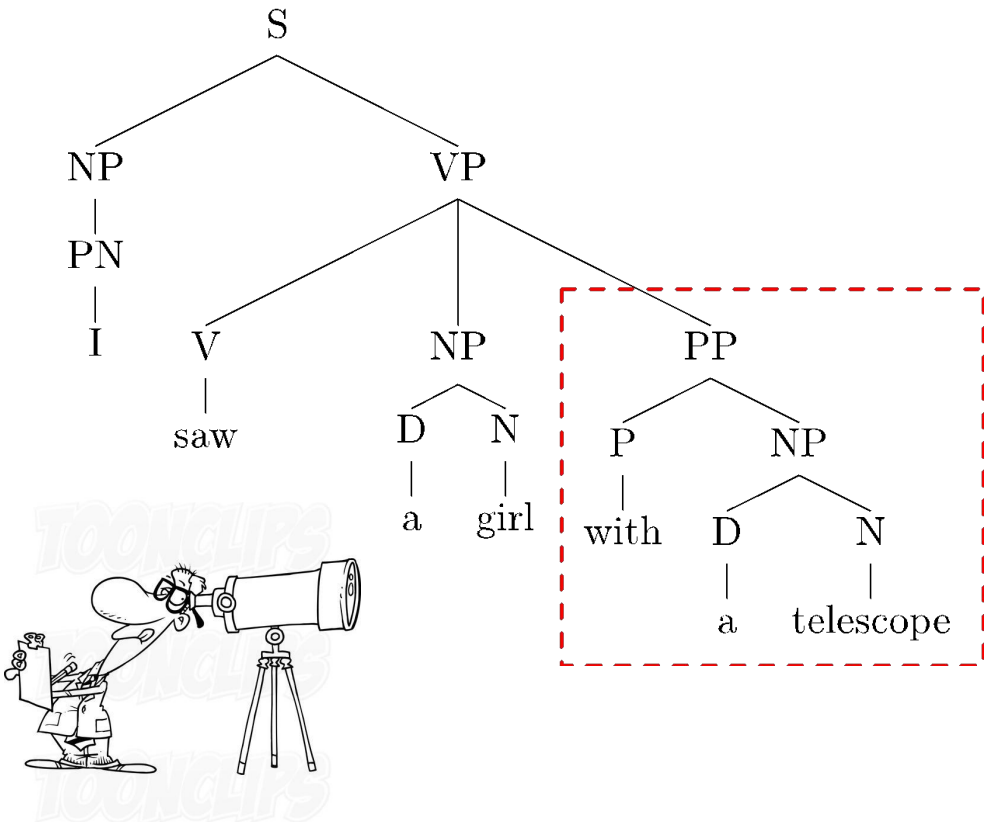
This tree would be ruled out if the context would be somehow captured (subject-verb agreement)

Ambiguities



Why parsing is hard? Ambiguity

- Prepositional phrase attachment ambiguity





PP Ambiguity

Put the block in the box on the table in the kitchen

- 3 prepositional phrases, 5 interpretations:
 - Put the block **((in the box on the table) in the kitchen)**
 - Put the block (in the box (on the table in the kitchen))
 - Put ((the block in the box) on the table) in the kitchen.
 - Put (the block (in the box on the table)) in the kitchen.
 - Put **(the block in the box) (on the table in the kitchen)**



PP Ambiguity

Put the block in the box on the table in the kitchen

- 3 prepositional phrases, 5 interpretations:
 - Put the block ((in the box on the table) in the kitchen)
 - Put the block (in the box (on the table in the kitchen))
 - Put ((the block in the box) on the table) in the kitchen.
 - Put (the block (in the box on the table)) in the kitchen.
 - Put (the block in the box) (on the table in the kitchen)

- A general case:

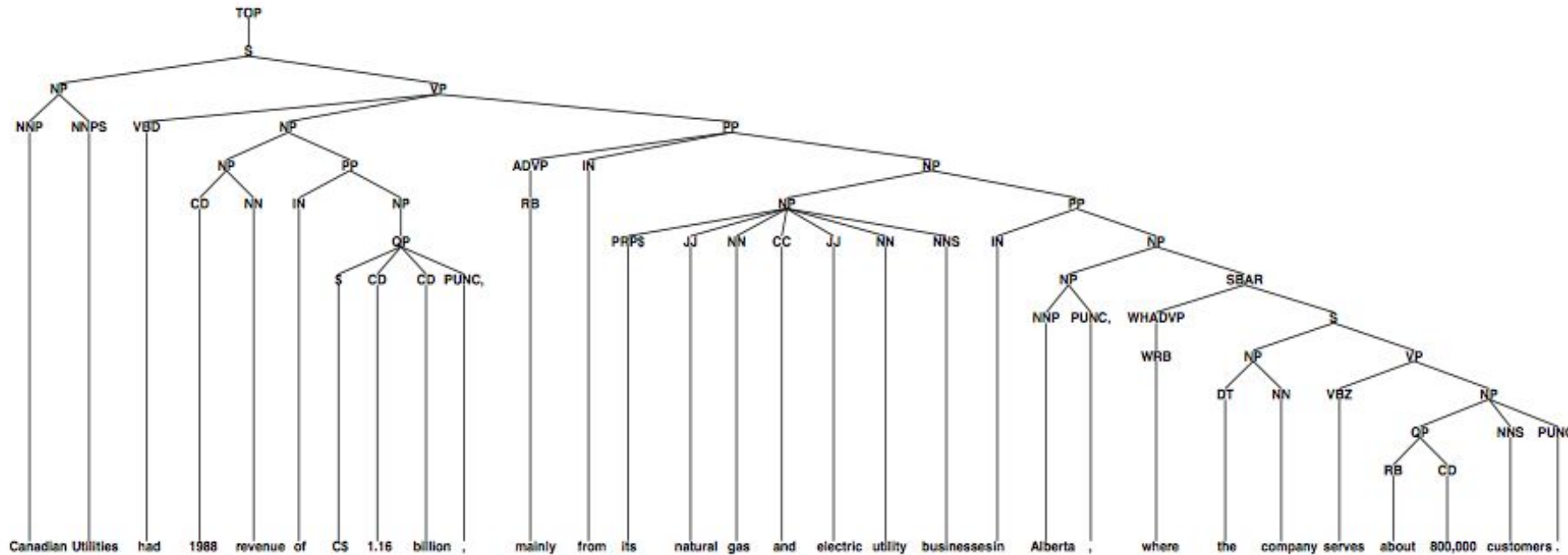
$$Cat_n = \binom{2n}{n} - \binom{2n}{n-1} \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$$

Catalan numbers

1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, ...



A typical tree from a standard dataset (Penn treebank WSJ)



Canadian Utilities had 1988 revenue of \$ 1.16 billion , mainly from its natural gas and electric utility businesses in Alberta , where the company serves about 800,000 customers .



Syntactic Ambiguities I

- Prepositional phrases:
They cooked the beans in the pot on the stove with handles.
- Particle vs. preposition:
The puppy tore up the staircase.
- Complement structures
The tourists objected to the guide that they couldn't hear.
She knows you like the back of her hand.
- Gerund vs. participial adjective
Visiting relatives can be boring.
Changing schedules frequently confused passengers.



Syntactic Ambiguities II

- **Modifier scope within NPs**
impractical design requirements
plastic cup holder
- **Multiple gap constructions**
The chicken is ready to eat.
The contractors are rich enough to sue.
- **Coordination scope:**
Small rats and mice can squeeze into holes or cracks in the wall.

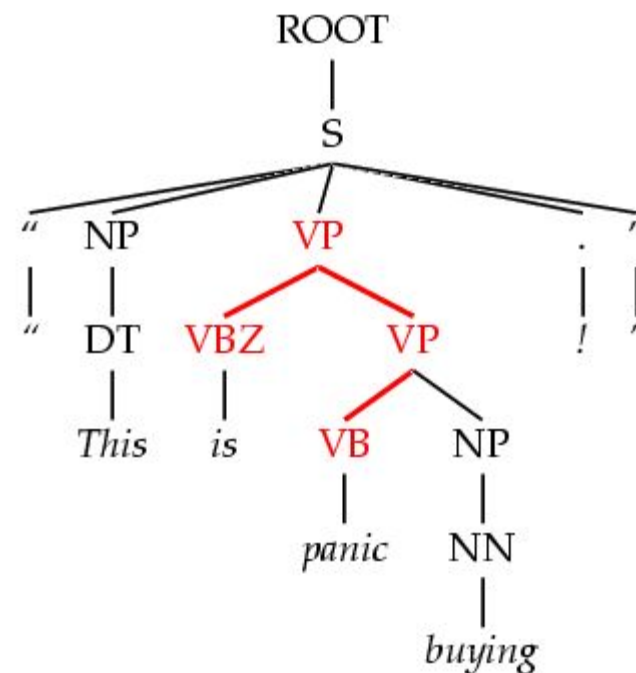


Dark Ambiguities

- *Dark ambiguities*: most analyses are shockingly bad (meaning, they don't have an interpretation you can get your mind around)

This analysis corresponds to the correct parse of

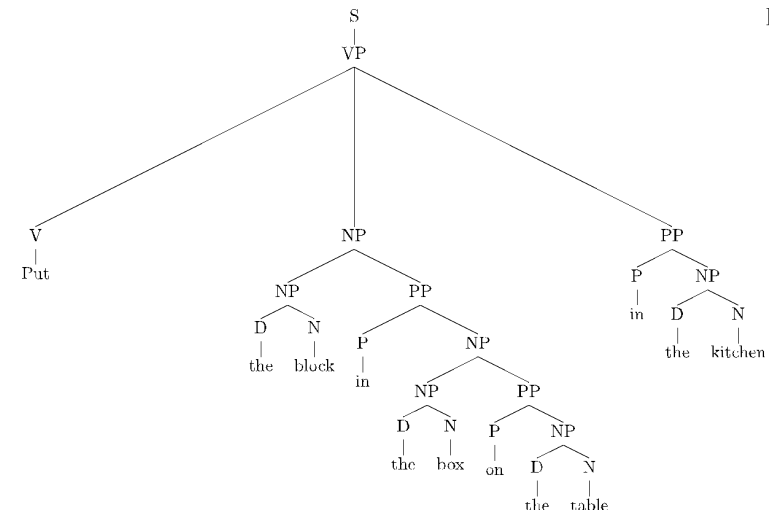
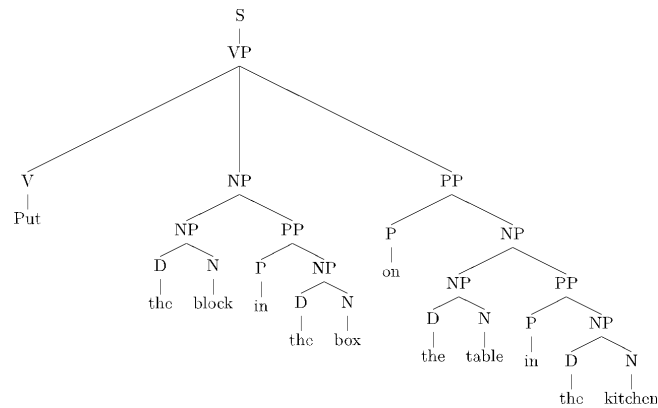
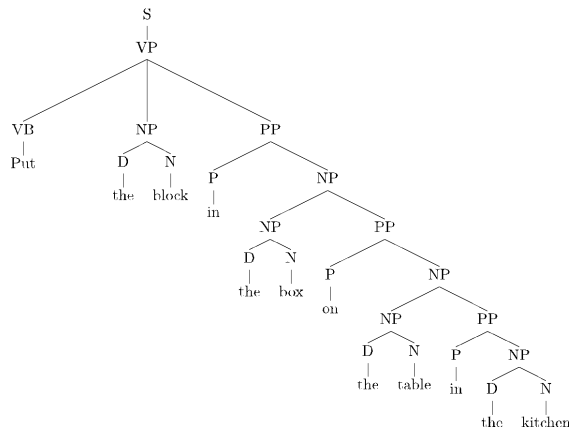
"This is panic buying !"



- Unknown words and new usages
- **Solution**: We need mechanisms to focus attention on the best ones, probabilistic techniques do this



How to Deal with Ambiguity?



Put the block in the box on the table in the kitchen

- We want to **score all the derivations** to encode how plausible they are

PCFGs



Probabilistic Context-Free Grammars

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 - Phrasal categories: S, NP, VP, ADJP, etc.
 - Parts-of-speech (pre-terminals): NN, JJ, DT, VB
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 - Of the form $X \rightarrow Y_1 Y_2 \dots Y_k$, with $X, Y_i \in N$
 - Examples: $S \rightarrow NP VP$, $VP \rightarrow VP CC VP$
 - Also called rewrites, productions, or local trees

- A PCFG adds:
 - A top-down production probability per rule $P(Y_1 Y_2 \dots Y_k \mid X)$



PCFGs

Associate probabilities with the rules : $p(X \rightarrow \alpha)$

$$\forall X \rightarrow \alpha \in R : 0 \leq p(X \rightarrow \alpha) \leq 1$$

$$\forall X \in N : \sum_{\alpha: X \rightarrow \alpha \in R} p(X \rightarrow \alpha) = 1$$

Now we can score a tree as a product of probabilities corresponding to the used rules

$S \rightarrow NP VP$	1.0	(NP A girl) (VP ate a sandwich)	$N \rightarrow girl$	0.2
$VP \rightarrow V$	0.2		$N \rightarrow telescope$	0.7
$VP \rightarrow V NP$	0.4	(VP ate) (NP a sandwich)	$N \rightarrow sandwich$	0.1
$VP \rightarrow VP PP$	0.4	(VP saw a girl) (PP with ...)	$PN \rightarrow I$	1.0
$NP \rightarrow NP PP$	0.3	(NP a girl) (PP with)	$V \rightarrow saw$	0.5
$NP \rightarrow D N$	0.5	(D a) (N sandwich)	$V \rightarrow ate$	0.5
$NP \rightarrow PN$	0.2		$P \rightarrow with$	0.6
$PP \rightarrow P NP$	1.0	(P with) (NP with a sandwich)	$P \rightarrow in$	0.4
			$D \rightarrow a$	0.3
			$D \rightarrow the$	0.7



PCFGs

S

$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

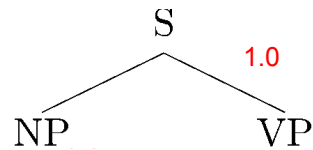
$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$p(T) =$



PCFGs



$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

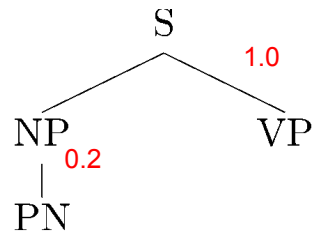
$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$p(T) = 1.0 \times$



PCFGs



$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

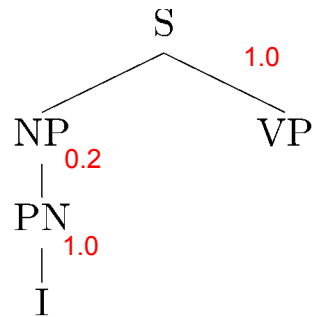
$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$$p(T) = 1.0 \times 0.2 \times$$



PCFGs



$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

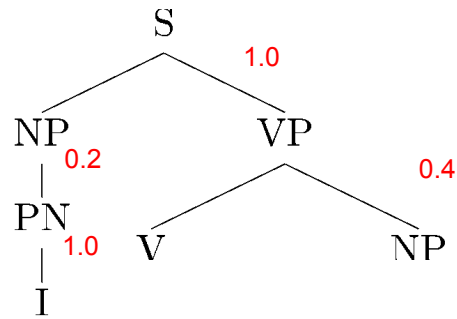
$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$$p(T) = 1.0 \times 0.2 \times 1.0 \times$$



PCFGs

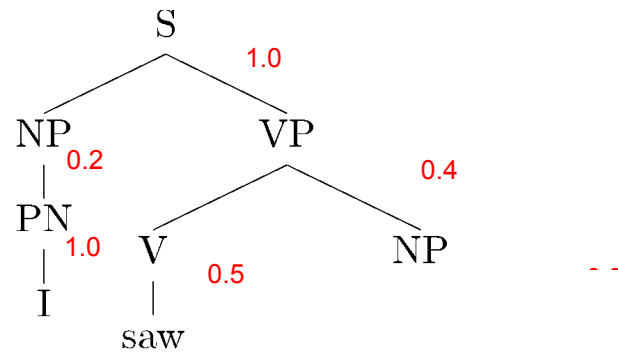


- $S \rightarrow NP VP$ 1.0
- $VP \rightarrow V$ 0.2
- $VP \rightarrow V NP$ 0.4
- $VP \rightarrow VP PP$ 0.4
- $NP \rightarrow NP PP$ 0.3
- $NP \rightarrow D N$ 0.5
- $NP \rightarrow PN$ 0.2
- $PP \rightarrow P NP$ 1.0
- $N \rightarrow girl$ 0.2
- $N \rightarrow telescope$ 0.7
- $N \rightarrow sandwich$ 0.1
- $PN \rightarrow I$ 1.0
- $V \rightarrow saw$ 0.5
- $V \rightarrow ate$ 0.5
- $P \rightarrow with$ 0.6
- $P \rightarrow in$ 0.4
- $D \rightarrow a$ 0.3
- $D \rightarrow the$ 0.7

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times$$



PCFGs



$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

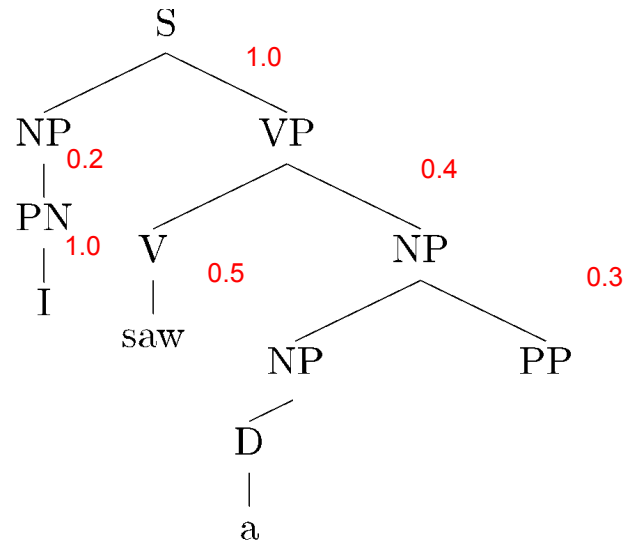
$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times$$



PCFGs



$S \rightarrow NP VP$ 1.0

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

$PP \rightarrow P NP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7

$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times$$



PCFGs

$S \rightarrow NP VP$ 1.0

$N \rightarrow girl$ 0.2

$VP \rightarrow V$ 0.2

$N \rightarrow telescope$ 0.7

$VP \rightarrow V NP$ 0.4

$N \rightarrow sandwich$ 0.1

$VP \rightarrow VP PP$ 0.4

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$NP \rightarrow NP PP$ 0.3

$V \rightarrow ate$ 0.5

$NP \rightarrow D N$ 0.5

$P \rightarrow with$ 0.6

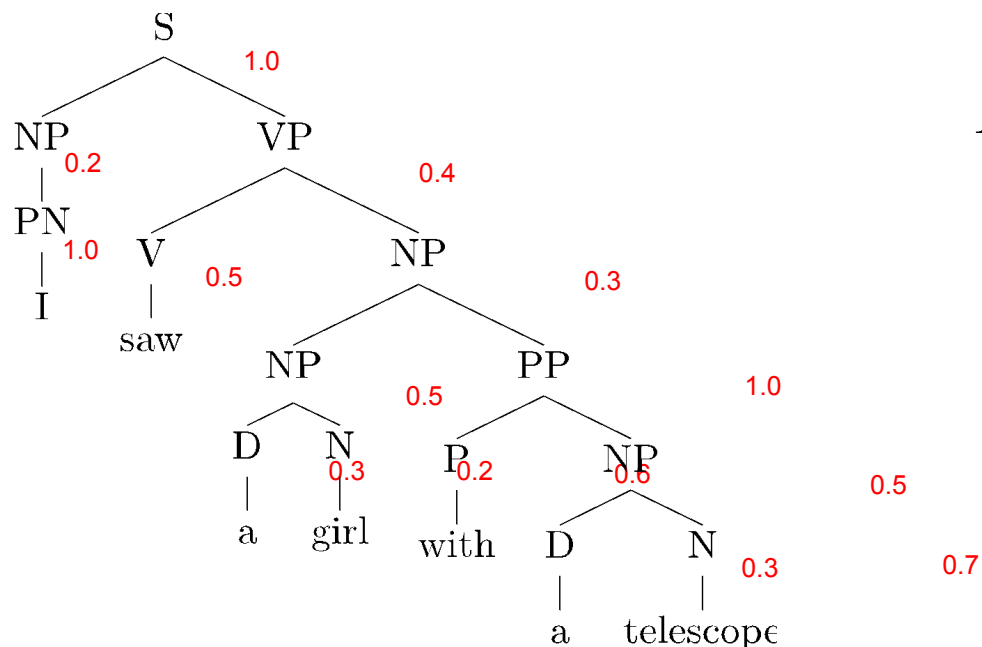
$NP \rightarrow PN$ 0.2

$P \rightarrow in$ 0.4

$PP \rightarrow P NP$ 1.0

$D \rightarrow a$ 0.3

$D \rightarrow the$ 0.7



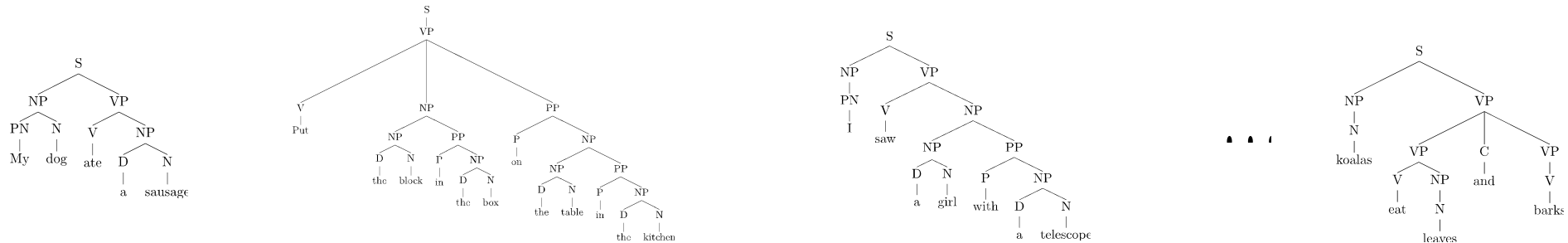
$$p(T) = 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 = 2.26 \times 10^{-5}$$

PCFG Estimation



ML estimation

- A treebank: a collection sentences annotated with constituent trees



- An estimated probability of a rule (maximum likelihood estimates)

$$p(X \rightarrow \alpha) = \frac{C(X \rightarrow \alpha)}{C(X)}$$

The number of times the rule used in the corpus

The number of times the nonterminal X appears in the treebank

- Smoothing is helpful
 - Especially important for preterminal rules



Distribution over trees

- We defined a distribution **over production rules for each nonterminal**
- Our goal was to define **a distribution over parse trees**

Unfortunately, not all PCFGs give rise to a proper distribution over trees, i.e. the sum over probabilities of all trees the grammar can generate may be less than 1: $\sum_T P(T) < 1$

- **Good news:** any PCFG estimated with the maximum likelihood procedure are always proper (Chi and Geman, 98)

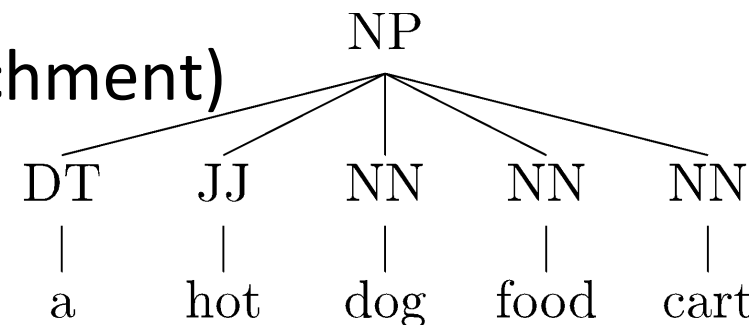


Penn Treebank: peculiarities

- Wall street journal: around 40, 000 annotated sentences, 1,000,000 words
 - Fine-grained part of speech tags (45), e.g., for verbs

VBD	Verb, past tense
VBG	Verb, gerund or present participle
VBP	Verb, present (non-3 rd person singular)
VBZ	Verb, present (3 rd person singular)
MD	Modal

- Flat NPs (no attempt to disambiguate NP attachment)



CKY Parsing



Parsing

- **Parsing is search** through the space of all possible parses
 - e.g., we may want either any parse, all parses or the highest scoring parse (if PCFG):

$$\arg \max_{T \in G(x)} P(T)$$

- **Bottom-up:**
 - One starts from words and attempt to construct the full tree
- **Top-down**
 - Start from the start symbol and attempt to expand to get the sentence



CKY algorithm (aka CYK)

- **Cocke-Kasami-Younger algorithm**
 - Independently discovered in late 60s / early 70s
- **An efficient bottom up parsing algorithm for (P)CFGs**
 - can be used both for the recognition and parsing problems
 - Very important in NLP (and beyond)
- **We will start with the non-probabilistic version**



Constraints on the grammar

- The basic CKY algorithm supports only rules in the **Chomsky Normal Form (CNF)**:

$$C \rightarrow x$$

Unary **preterminal** rules (generation of words given PoS tags)

$$N \rightarrow \text{telescope} \quad D \rightarrow \text{the}$$

$$C \rightarrow C_1 C_2$$

Binary **inner** rules $S \rightarrow NP VP$ $NP \rightarrow D N$



Constraints on the grammar

- The basic CKY algorithm supports only rules in the **Chomsky Normal Form (CNF)**:

$$C \rightarrow x$$

$$C \rightarrow C_1 C_2$$

- Any CFG can be converted to an equivalent CNF
 - Equivalent means that they define **the same language**
 - However (syntactic) **trees will look differently**
 - It is possible to address it by defining such transformations that allows for easy **reverse transformation**



Transformation to CNF form

- What one need to do to convert to CNF form

- Get rid of unary rules:

$$C \rightarrow C_1$$

- Get rid of N-ary rules: $C \rightarrow C_1 C_2 \dots C_n \quad (n > 2)$

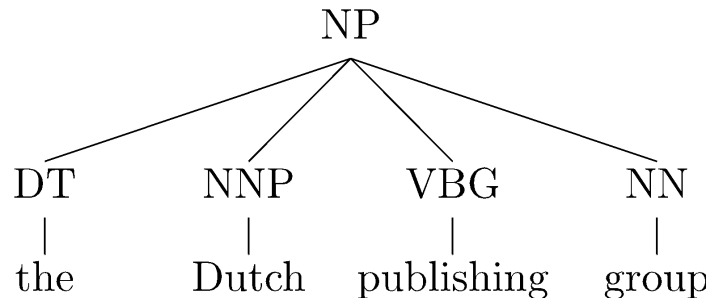
Not a problem, as our CKY algorithm will support unary rules

Crucial to process them, as required for efficient parsing



Transformation to CNF form: binarization

- Consider $NP \rightarrow DT\ NNP\ VBG\ NN$

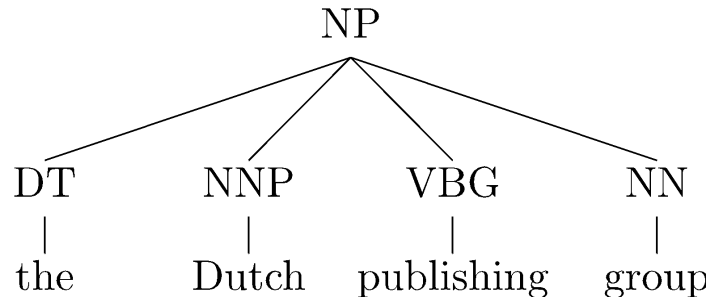


- How do we get a set of binary rules which are equivalent?



Transformation to CNF form: binarization

- Consider $NP \rightarrow DT\ NNP\ VBG\ NN$



- How do we get a set of binary rules which are equivalent?

$NP \rightarrow DT\ X$

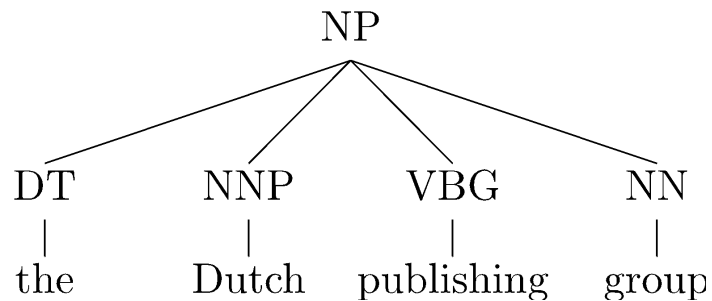
$X \rightarrow NNP\ Y$

$Y \rightarrow VBG\ NN$



Transformation to CNF form: binarization

- Consider $NP \rightarrow DT\ NNP\ VBG\ NN$



- How do we get a set of binary rules which are equivalent?

$NP \rightarrow DT\ X$

$X \rightarrow NNP\ Y$

$Y \rightarrow VBG\ NN$

- A more systematic way to refer to new non-terminals

$NP \rightarrow DT\ @NP|DT$

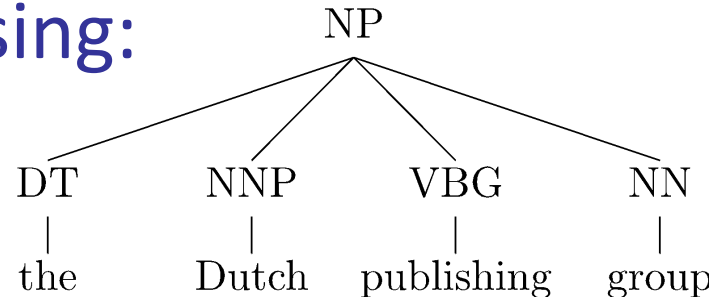
$@NP|DT \rightarrow NNP\ @NP|DT_NNP$

$@NP|DT_NNP \rightarrow VBG\ NN$

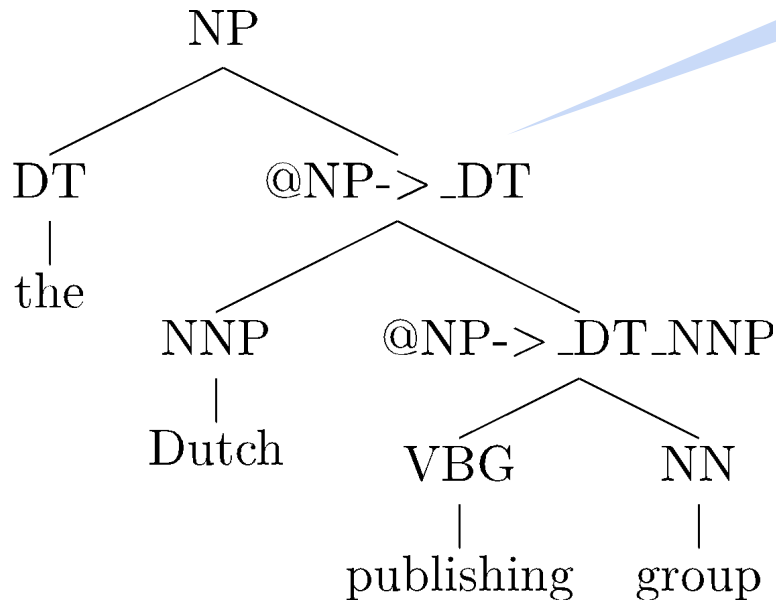


Transformation to CNF form: binarization

- Instead of binarizing tuples we can binarize trees on preprocessing:



Also known as **lossless Markovization** in the context of PCFGs



Can be easily reversed on postprocessing



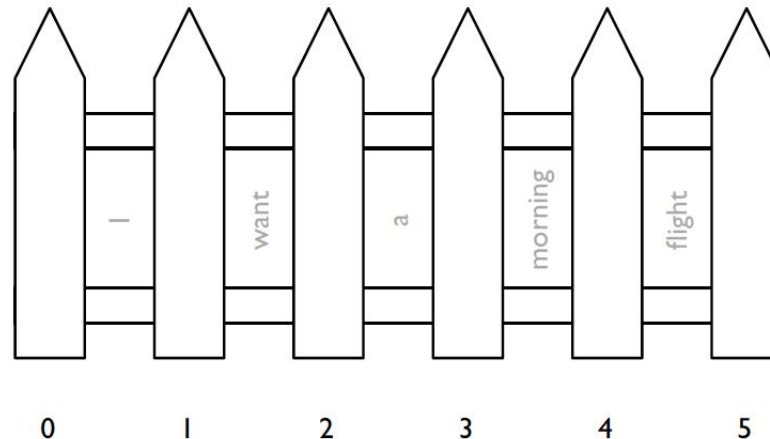
CKY: Parsing task

- We are given
 - a grammar $\langle N, T, S, R \rangle$
 - a sequence of words $w = (w_1, w_2, \dots, w_n)$
- Our goal is to produce a parse tree for w



CKY: Parsing task

- We are given
 - a grammar $\langle N, T, S, R \rangle$
 - a sequence of words $w = (w_1, w_2, \dots, w_n)$
- Our goal is to produce a parse tree for w
- We need an easy way to refer to substrings of w



indices refer to fenceposts

span (i, j) refers to words between fenceposts i and j



Parsing one word

$$C \rightarrow w_i$$

w_i



Parsing one word

$$C \rightarrow w_i$$

C

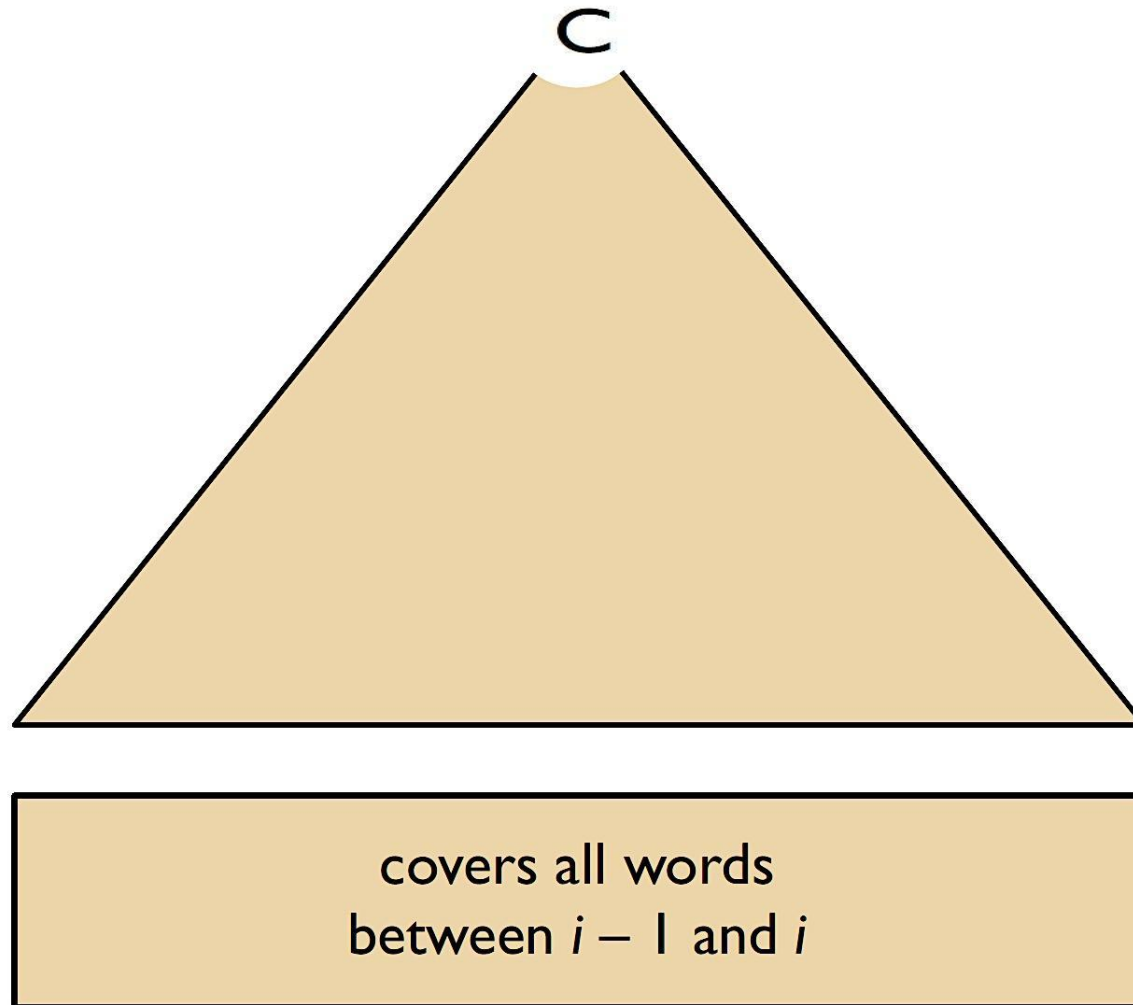


w_i



Parsing one word

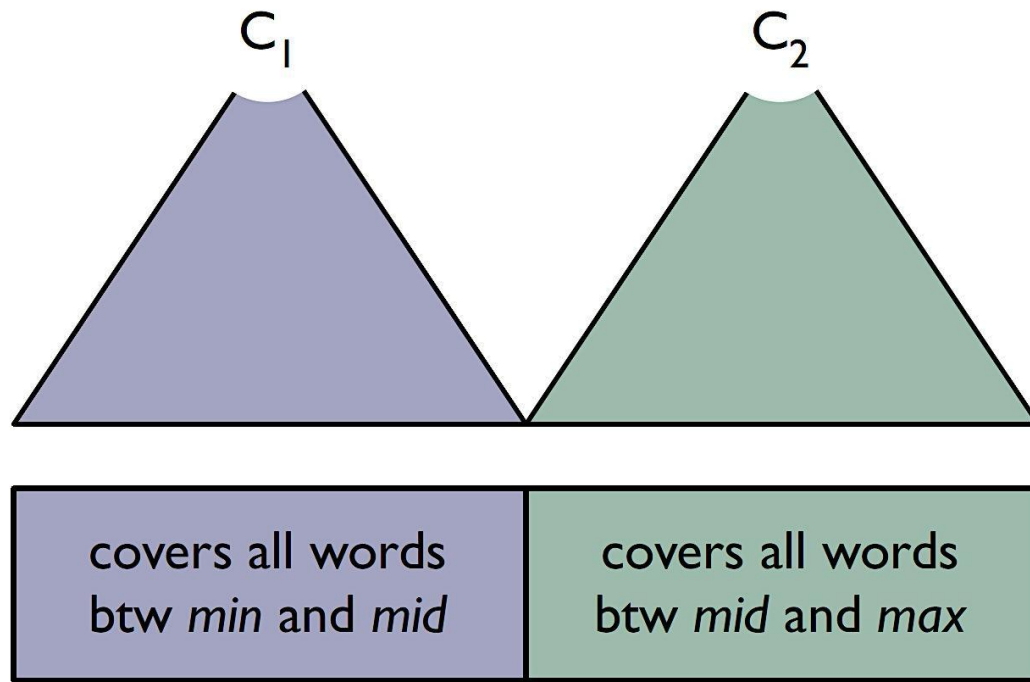
$$C \rightarrow w_i$$





Parsing longer spans

$$C \rightarrow C_1 C_2$$

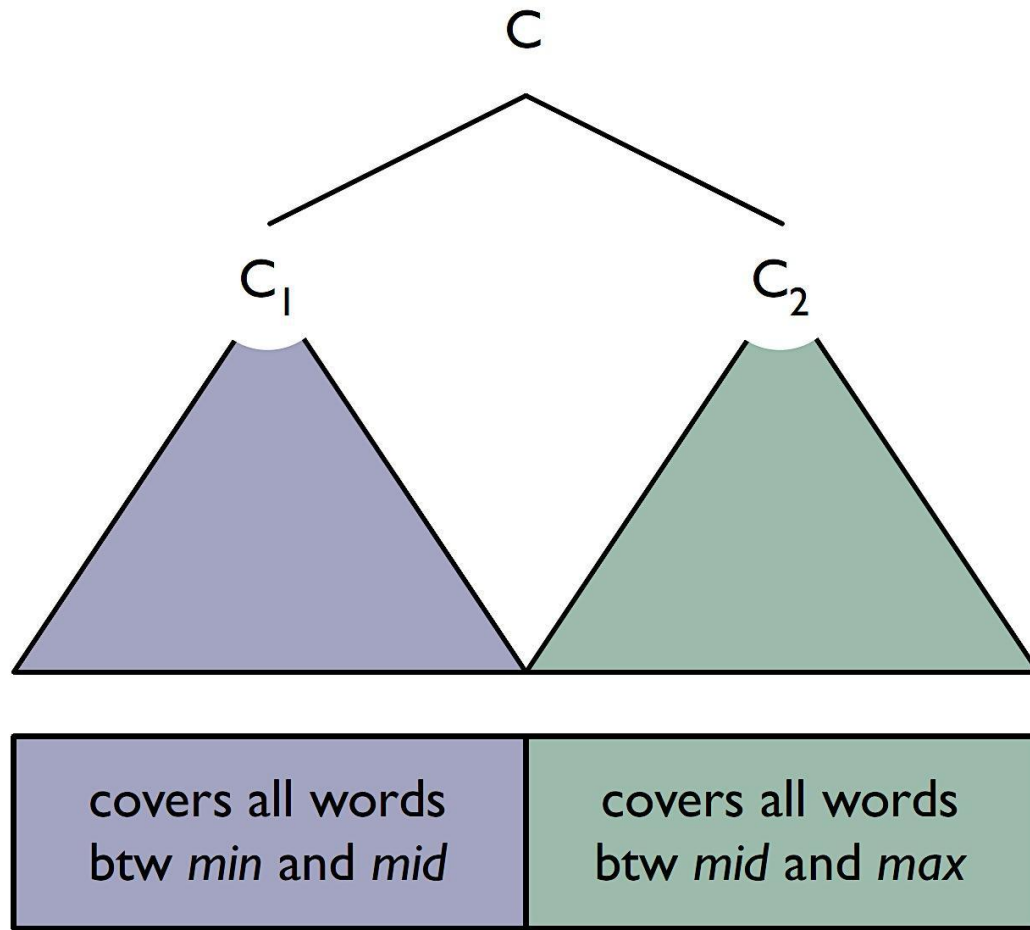


Check through all C_1 , C_2 , mid



Parsing longer spans

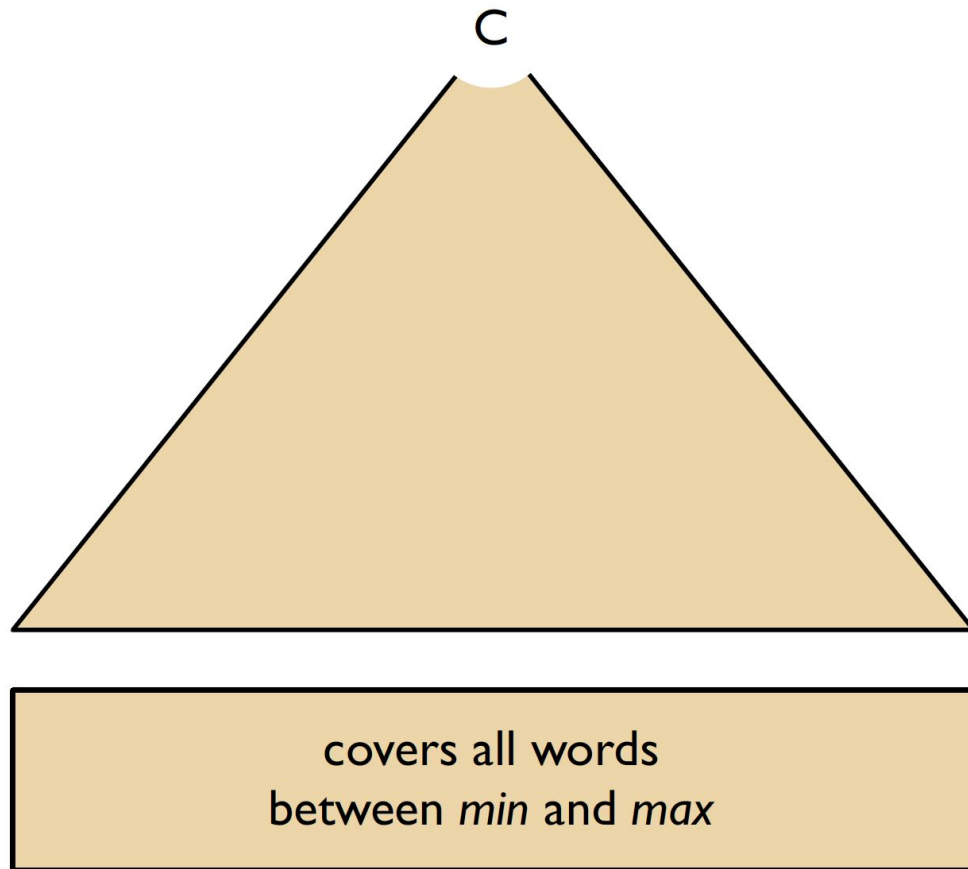
$$C \rightarrow C_1 C_2$$



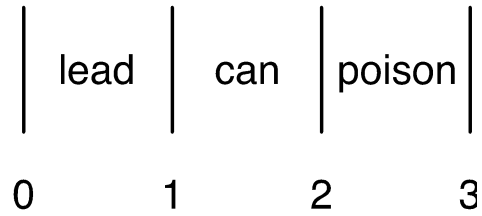
Check through all
 C_1 , C_2 , mid



Parsing longer spans



CKY in action



$$S \rightarrow NP VP$$

$$VP \rightarrow M V$$

$$VP \rightarrow V$$

$$NP \rightarrow N$$

$$NP \rightarrow N NP$$

$$N \rightarrow can$$

$$N \rightarrow lead$$

$$N \rightarrow poison$$

$$M \rightarrow can$$

$$M \rightarrow must$$

$$V \rightarrow poison$$

$$V \rightarrow lead$$

Inner rules

Preterminal rules

lead	can	poison
0	1	2

max = 1 max = 2 max = 3

min = 0			S?
min = 1			
min = 2			

Chart (aka parsing triangle)

$S \rightarrow NP VP$

$VP \rightarrow M V$
 $VP \rightarrow V$

$NP \rightarrow N$
 $NP \rightarrow N NP$

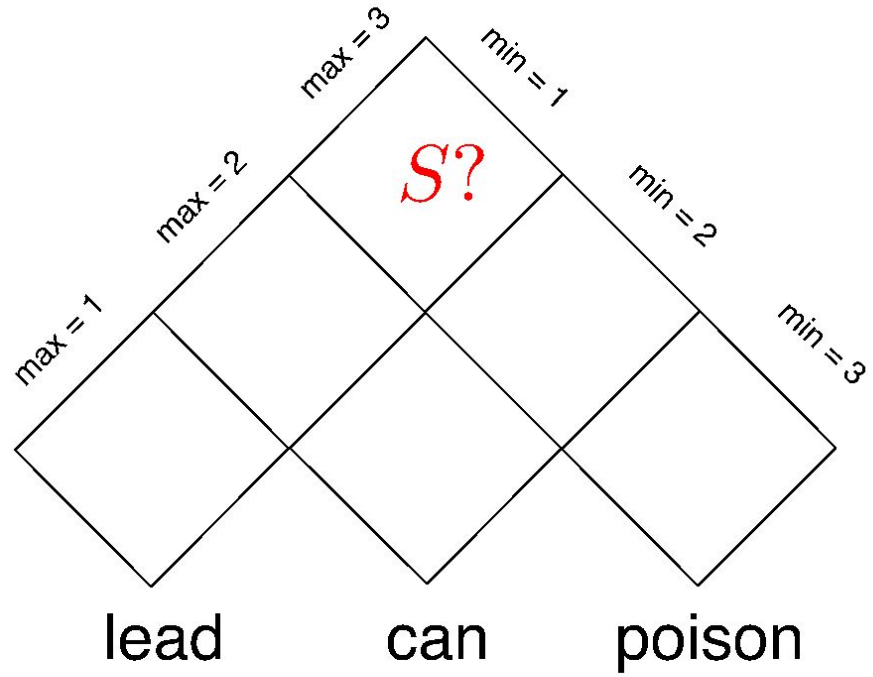
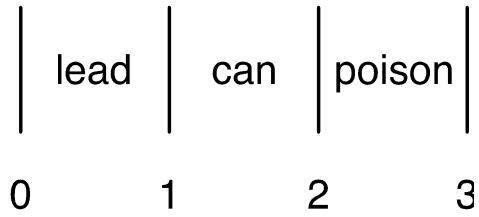
$N \rightarrow can$
 $N \rightarrow lead$
 $N \rightarrow poison$

$M \rightarrow can$
 $M \rightarrow must$

$V \rightarrow poison$
 $V \rightarrow lead$

Inner rules

Preterminal rules



$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

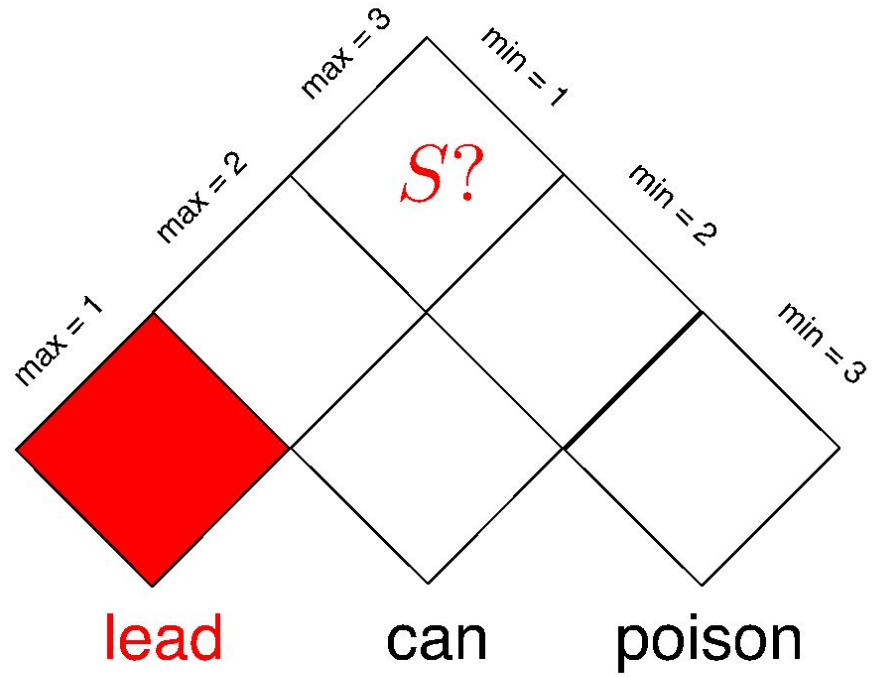
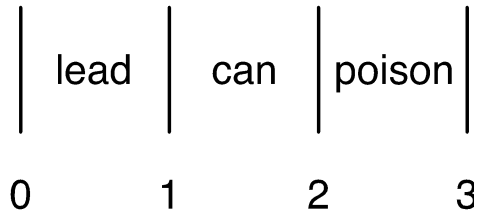
$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules



$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

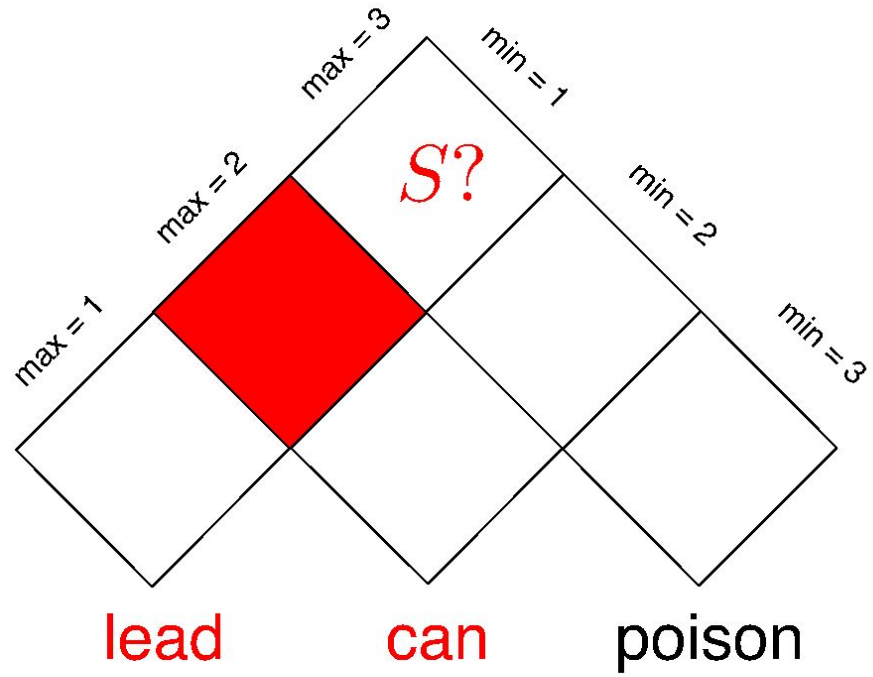
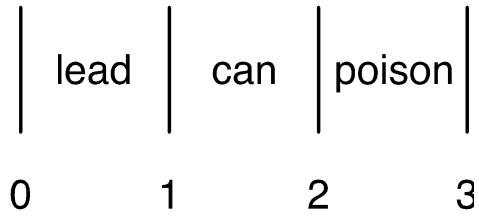
$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules



$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

| lead | can | poison |
 0 1 2 3

max = 1 max = 2 max = 3

min = 0			<i>S?</i>
min = 1			
min = 2			

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

| lead | can | poison |
 0 1 2 3

max = 1 max = 2 max = 3

min = 0	1	4	6 <i>S?</i>
min = 1		2	5
min = 2			3

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

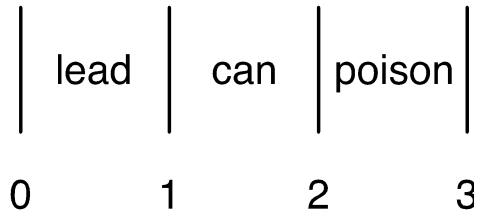
$M \rightarrow must$

$V \rightarrow poison$

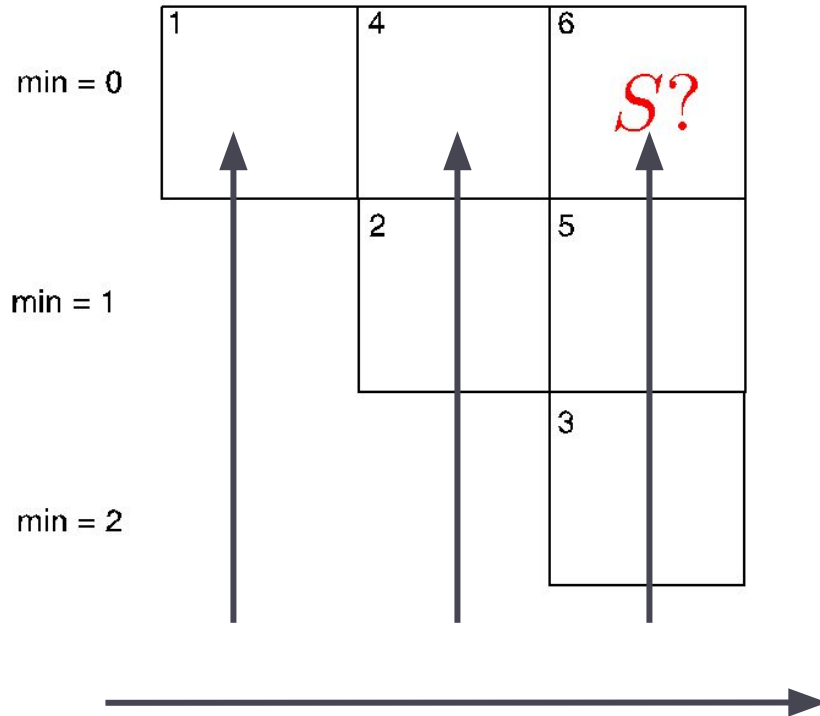
$V \rightarrow lead$

Inner rules

Preterminal rules



max = 1 max = 2 max = 3



$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

Inner rules

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

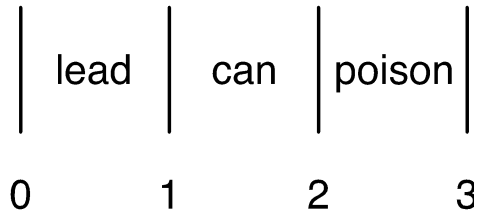
$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules



max = 1 max = 2 max = 3

min = 0	1 ?		
min = 1		2 ?	
min = 2			3 ?

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

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$V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2
1	2	3

	max = 1	max = 2	max = 3
min = 0	1 ?		
min = 1		2 ?	
min = 2			3 ?

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

- $N \rightarrow can$
 - $N \rightarrow lead$
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 - $M \rightarrow can$
 - $M \rightarrow must$

 - $V \rightarrow poison$
 - $V \rightarrow lead$

Inner rules

Preterminal rules

| lead | can | poison |
 0 1 2 3

		max = 1	max = 2	max = 3
min = 0	1 <i>N, V</i>			
min = 1		2 <i>N, M</i>		
min = 2			3 <i>N, V</i>	

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

-
- $N \rightarrow can$
 - $N \rightarrow lead$
 - $N \rightarrow poison$

 - $M \rightarrow can$
 - $M \rightarrow must$

 - $V \rightarrow poison$
 - $V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2

Check about unary rules

max = 1 max = 2 max = 3

min = 0	¹ <i>N, V</i> <i>NP, VP</i>		
min = 1		² <i>N, M</i> <i>NP</i>	
min = 2			³ <i>N, V</i> <i>NP, VP</i>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

Inner rules

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Preterminal rules

| lead | can | poison |
 0 1 2 3

max = 1 max = 2 max = 3

min = 0	¹ <i>N, V</i> <i>NP, VP</i>	⁴ ?	
min = 1		² <i>N, M</i> <i>NP</i>	
min = 2			³ <i>N, V</i> <i>NP, VP</i>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2

max = 1 max = 2 max = 3

min = 0	<div style="display: flex; justify-content: space-between;"> 1 4 </div> <div style="text-align: center; color: blue;"> N, V NP, VP </div> <div style="text-align: center; color: red; font-size: 2em;">?</div>	
min = 1	<div style="display: flex; justify-content: space-between;"> 2 </div> <div style="text-align: center; color: blue;"> N, M NP </div>	
min = 2		<div style="display: flex; justify-content: space-between;"> 3 </div> <div style="text-align: center; color: blue;"> N, V NP, VP </div>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

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$N \rightarrow lead$

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Inner rules

Preterminal rules

lead	can	poison
0	1	2

max = 1 max = 2 max = 3

min = 0	<div style="display: flex; justify-content: space-between;"> 1 4 </div> <div style="text-align: center;"> <i>N, V</i> <i>NP, VP</i> </div>	<div style="display: flex; justify-content: space-between;"> </div>
min = 1	<div style="display: flex; justify-content: space-between;"> 2 </div> <div style="text-align: center;"> <i>N, M</i> <i>NP</i> </div>	<div style="display: flex; justify-content: space-between;"> </div>
min = 2	<div style="display: flex; justify-content: space-between;"> </div>	<div style="display: flex; justify-content: space-between;"> 3 </div> <div style="text-align: center;"> <i>N, V</i> <i>NP, VP</i> </div>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

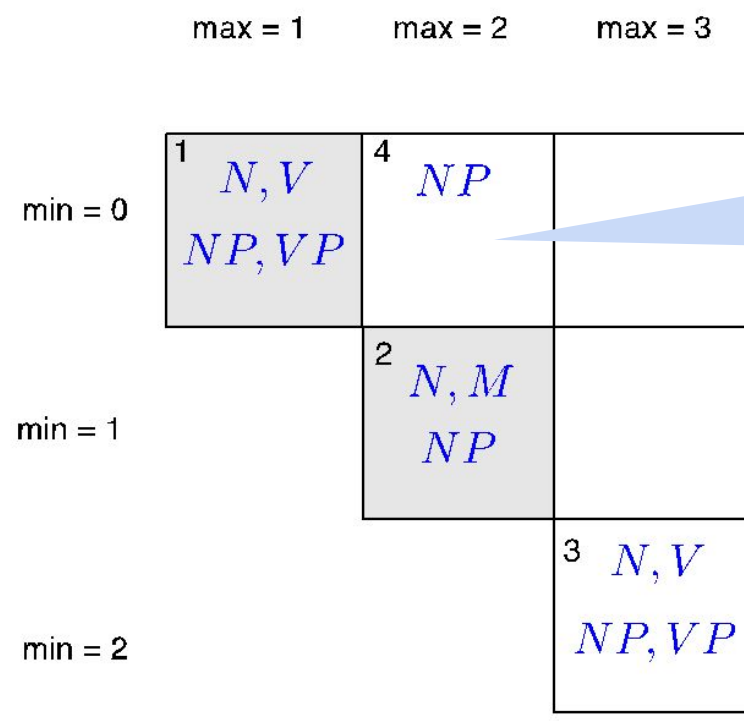
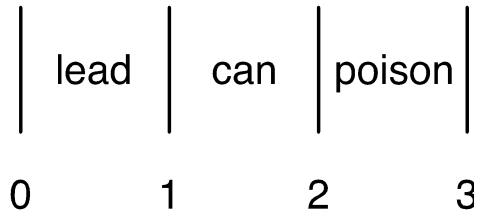
$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules



Check about unary rules: no unary rules here

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2

max = 1 max = 2 max = 3

min = 0	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> ¹ <i>N, V</i> <i>NP, VP</i> </div>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> ⁴ <i>NP</i> </div>	
min = 1		<div style="border: 1px solid black; padding: 5px; display: inline-block;"> ² <i>N, M</i> <i>NP</i> </div>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> ⁵ ? </div>
min = 2			<div style="border: 1px solid black; padding: 5px; display: inline-block;"> ³ <i>N, V</i> <i>NP, VP</i> </div>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

lead	can	poison
0	1	2

max = 1 max = 2 max = 3

min = 0	¹ <i>N, V</i> <i>NP, VP</i>	⁴ <i>NP</i>	
min = 1		² <i>N, M</i> <i>NP</i>	⁵ <i>S, VP,</i> <i>NP</i>
min = 2			³ <i>N, V</i> <i>NP, VP</i>

$S \rightarrow NP VP$

$VP \rightarrow M V$
 $VP \rightarrow V$

$NP \rightarrow N$
 $NP \rightarrow N NP$

N → can
N → lead
N → poison

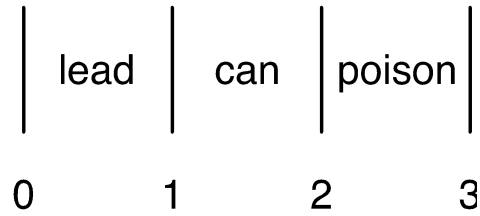
M → can
M → must

V → poison
V → lead

Inner rules

Preterminal rules

CKY in action



		max = 1	max = 2	max = 3
min = 0	¹ <i>N, V</i> <i>NP, VP</i>	⁴ <i>NP</i>		
min = 1		² <i>N, M</i> <i>NP</i>	⁵ <i>S, VP,</i> <i>NP</i>	
min = 2			³ <i>N, V</i> <i>NP, VP</i>	

Check about unary rules: no unary rules here

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

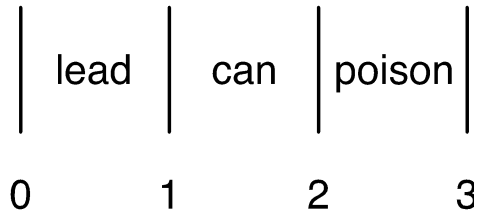
$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules



max = 1 max = 2 max = 3

min = 0	<div style="display: flex; justify-content: space-between;"> 1 <i>N, V</i> </div> <div style="display: flex; justify-content: space-between;"> <i>NP, VP</i> </div>	<div style="display: flex; justify-content: space-between;"> 4 <i>NP</i> </div>	<div style="display: flex; justify-content: space-between;"> 6 ? </div>
min = 1	<div style="display: flex; justify-content: space-between;"> 2 <i>N, M</i> </div> <div style="display: flex; justify-content: space-between;"> <i>NP</i> </div>	<div style="display: flex; justify-content: space-between;"> 5 <i>S, VP,</i> </div> <div style="display: flex; justify-content: space-between;"> <i>NP</i> </div>	
min = 2		<div style="display: flex; justify-content: space-between;"> 3 <i>N, V</i> </div> <div style="display: flex; justify-content: space-between;"> <i>NP, VP</i> </div>	

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

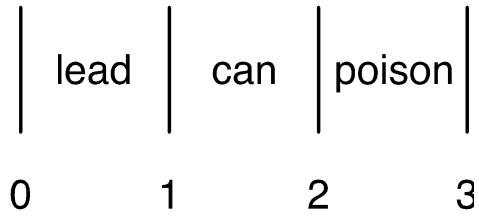
$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules



max = 1 max = 2 max = 3

min = 0	1 <i>N, V</i> <hr style="border: 1px solid red;"/> <i>NP, VP</i>	4 <i>NP</i>	6 ?
min = 1		2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i> <hr style="border: 1px solid red;"/>
min = 2			3 <i>N V</i> <i>NP VP</i> <hr style="border: 1px solid red;"/>

$S \rightarrow NP VP$

$VP \rightarrow M V$

$VP \rightarrow V$

$NP \rightarrow N$

$NP \rightarrow N NP$

$N \rightarrow can$

$N \rightarrow lead$

$N \rightarrow poison$

$M \rightarrow can$

$M \rightarrow must$

$V \rightarrow poison$

$V \rightarrow lead$

Inner rules

Preterminal rules

| lead | can | poison |
 0 1 2 3

max = 1 max = 2 max = 3

min = 0	1 N, V NP, VP	4 NP	6 S, NP
min = 1		2 N, M NP	5 S, VP, NP
min = 2			3 N, V NP, VP

mid=1

$S \rightarrow NP VP$

$VP \rightarrow M V$
 $VP \rightarrow V$

$NP \rightarrow N$
 $NP \rightarrow N NP$

$N \rightarrow can$
 $N \rightarrow lead$
 $N \rightarrow poison$

$M \rightarrow can$
 $M \rightarrow must$

$V \rightarrow poison$
 $V \rightarrow lead$

Inner rules

Preterminal rules

$S \rightarrow NP VP$

| lead | can | poison |
0 1 2 3

max = 1 max = 2 max = 3

mid=2

min = 0	1 N, V NP, VP	4 NP	6 S, NP $S(?!)$
min = 1		2 N, M NP	5 $S, VP,$ NP
min = 2			3 N, V NP, VP

$VP \rightarrow M V$
 $VP \rightarrow V$

$NP \rightarrow N$
 $NP \rightarrow N NP$

$N \rightarrow can$
 $N \rightarrow lead$
 $N \rightarrow poison$

$M \rightarrow can$
 $M \rightarrow must$

$V \rightarrow poison$
 $V \rightarrow lead$

Inner rules

Preterminal rules

$S \rightarrow NP VP$

| lead | can | poison |
 0 1 2 3

max = 1 max = 2 max = 3

min = 0	1 N, V NP, VP	4 NP	6 S, NP $S(?!)$
min = 1		2 N, M NP	5 $S, VP,$ NP
min = 2			3 N, V NP, VP

$VP \rightarrow M V$
 $VP \rightarrow V$

$NP \rightarrow N$
 $NP \rightarrow N NP$

$N \rightarrow can$
 $N \rightarrow lead$
 $N \rightarrow poison$

$M \rightarrow can$
 $M \rightarrow must$

$V \rightarrow poison$
 $V \rightarrow lead$

Inner rules

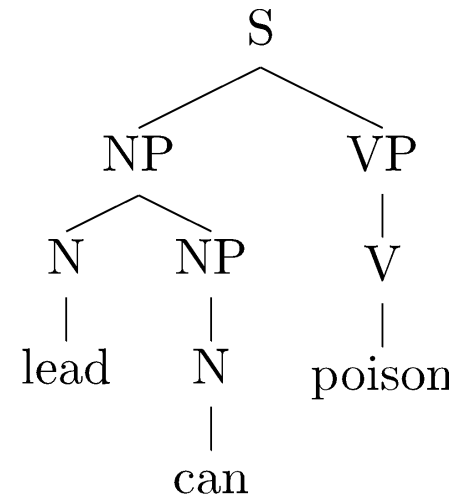
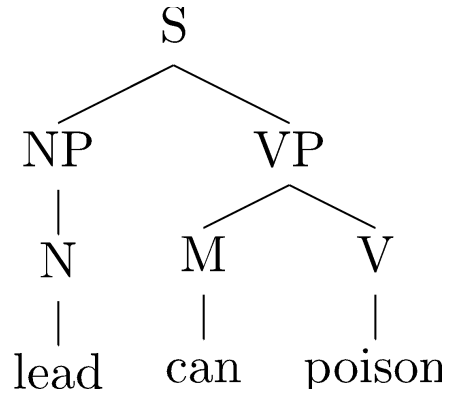
Preterminal rules

Apparently the sentence is ambiguous for the grammar: (as the grammar overgenerates)



Ambiguity

-



No subject-verb agreement, and *poison* used as an intransitive verb



CKY more formally

Here we assume that labels (C) are integer indices

Chart can be represented by a Boolean 3D array $\text{chart}[\text{min}][\text{max}][C]$

- ▶ Relevant entries have $0 < \text{min} < \text{max} \leq n$

$\text{chart}[\text{min}][\text{max}][C] = \text{true}$ if the signature $(\text{min}, \text{max}, C)$ is already added to the chart;
 false otherwise.

	max = 1	max = 2	max = 3
min = 0	1 <i>N, V</i> <i>NP, VP</i>	4 <i>NP</i>	6 <i>S, VP,</i> <i>NP</i>
min = 1		2 <i>N, M</i> <i>NP</i>	5 <i>S, VP,</i> <i>NP</i>
min = 2			3 <i>N, V</i> <i>NP, VP</i>



Implementation: preterminal rules

```
for each  $w_i$  from left to right
```

```
  for each preterminal rule  $C \rightarrow w_i$ 
```

```
    chart[i - 1][i][C] = true
```




Implementation: binary rules

```
for each max from 2 to n
```

```
  for each min from max - 2 down to 0
```

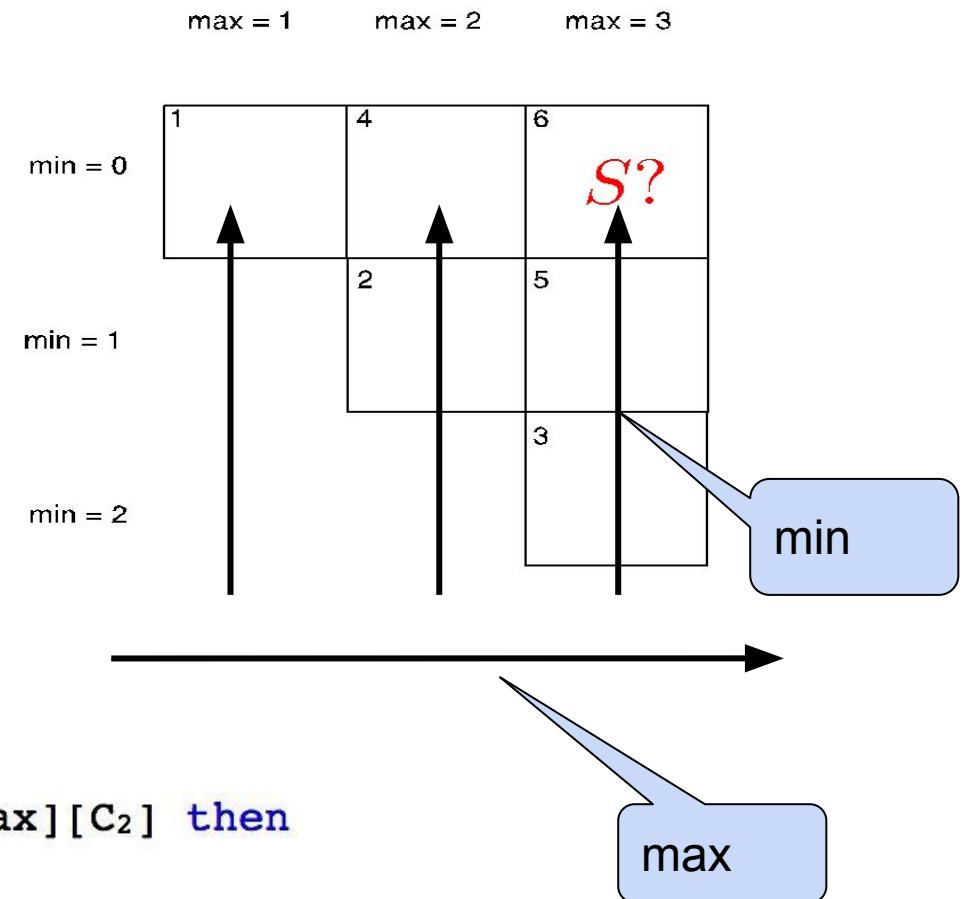
```
    for each syntactic category C
```

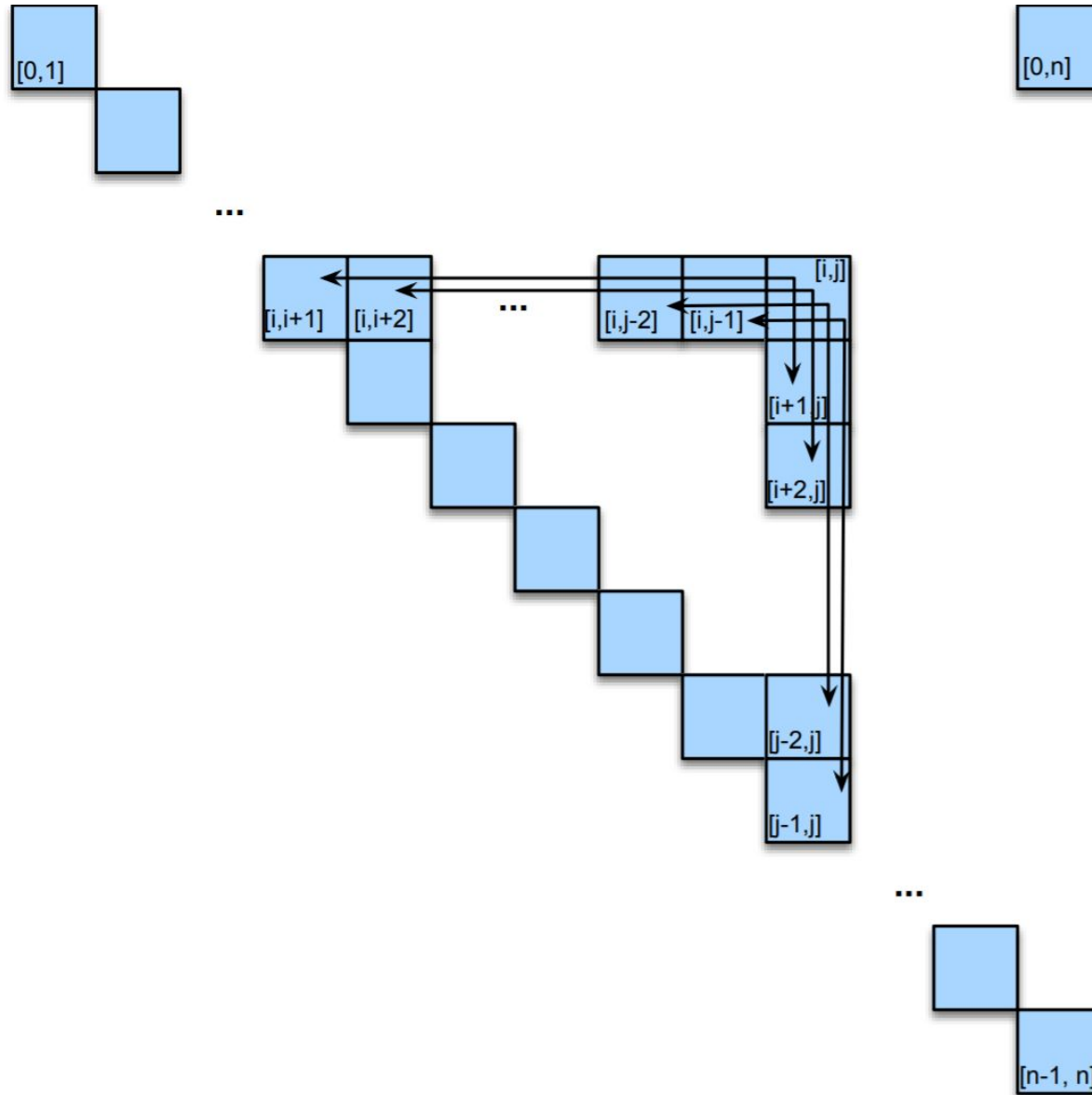
```
      for each binary rule  $C \rightarrow C_1 C_2$ 
```

```
        for each mid from min + 1 to max - 1
```

```
          if  $\text{chart}[\text{min}][\text{mid}][C_1]$  and  $\text{chart}[\text{mid}][\text{max}][C_2]$  then
```

```
             $\text{chart}[\text{min}][\text{max}][C] = \text{true}$ 
```







Unary rules

- How to integrate unary rules $C \rightarrow C_1$?



Unary rules

- How to integrate unary rules $C \rightarrow C_1$?

```
for each max from 1 to n
  for each min from max - 1 down to 0
    // First, try all binary rules as before.
    ...
    // Then, try all unary rules.
    for each syntactic category C
      for each unary rule C -> C1
        if chart[min][max][C1] then
          chart[min][max][C] = true
```

new bounds!



Unary rules

- How to integrate unary rules $C \rightarrow C_1$?

```
for each max from 1 to n
  for each min from max - 1 down to 0
    // First, try all binary rules as before.
    ...
    // Then, try all unary rules.
    for each syntactic category C
      for each unary rule C -> C1
        if chart[min][max][C1] then
```

new bounds!

But we forgot something!



Unary closure

- What if the grammar contained 2 rules:

$$A \rightarrow B$$

$$B \rightarrow C$$

- But C can be derived from A by a chain of rules:

$$A \rightarrow B \rightarrow C$$

- One could support chains in the algorithm but it is easier to extend the grammar, to get the **transitive closure**

$$\begin{array}{l} A \rightarrow B \\ B \rightarrow C \end{array} \quad \Rightarrow \quad \begin{array}{l} A \rightarrow B \\ B \rightarrow C \\ A \rightarrow C \end{array}$$



Unary closure

- What if the grammar contained 2 rules:

$$A \rightarrow B$$

$$B \rightarrow C$$

- But C can be derived from A by a chain of rules:

$$A \rightarrow B \rightarrow C$$

- One could support chains in the algorithm but it is easier to extend the grammar, to get the **transitive closure**

$$A \rightarrow B$$

$$B \rightarrow C$$

$$\Rightarrow$$

$$A \rightarrow B$$

$$B \rightarrow C$$

$$A \rightarrow C$$

$$A \rightarrow A$$

$$B \rightarrow B$$

$$C \rightarrow C$$

Convenient for programming reasons in the PCFG case



Algorithm analysis

Time complexity?

```
for each max from 2 to n
```

```
  for each min from max - 2 down to 0
```

```
    for each syntactic category C
```

```
      for each binary rule  $C \rightarrow C_1 C_2$ 
```

```
        for each mid from min + 1 to max - 1
```




Algorithm analysis

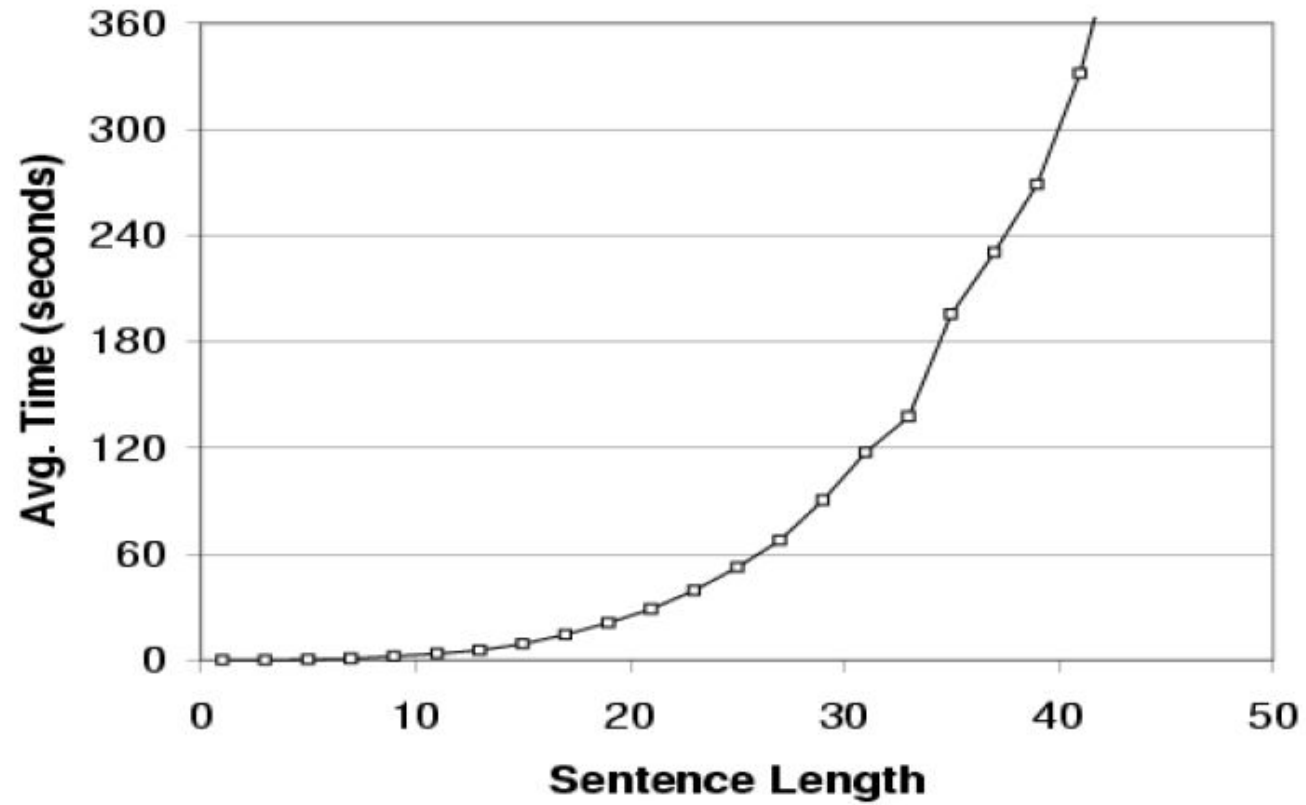
Time complexity?

```
for each max from 2 to n
  for each min from max - 2 down to 0
    for each syntactic category C
      for each binary rule C -> C1 C2
        for each mid from min + 1 to max - 1
```

$O(n^3 |R|)$ where $|R|$ is the number of rules in the grammar



Practical time complexity



$$\sim n^{3.6}$$



Probabilistic CKY

PCFGs

$S \rightarrow NP VP$ 1.0

$N \rightarrow girl$ 0.2

$N \rightarrow telescope$ 0.7

$VP \rightarrow V$ 0.2

$VP \rightarrow V NP$ 0.4

$VP \rightarrow VP PP$ 0.4

$N \rightarrow sandwich$ 0.1

$PN \rightarrow I$ 1.0

$V \rightarrow saw$ 0.5

$V \rightarrow ate$ 0.5

$NP \rightarrow NP PP$ 0.3

$NP \rightarrow D N$ 0.5

$NP \rightarrow PN$ 0.2

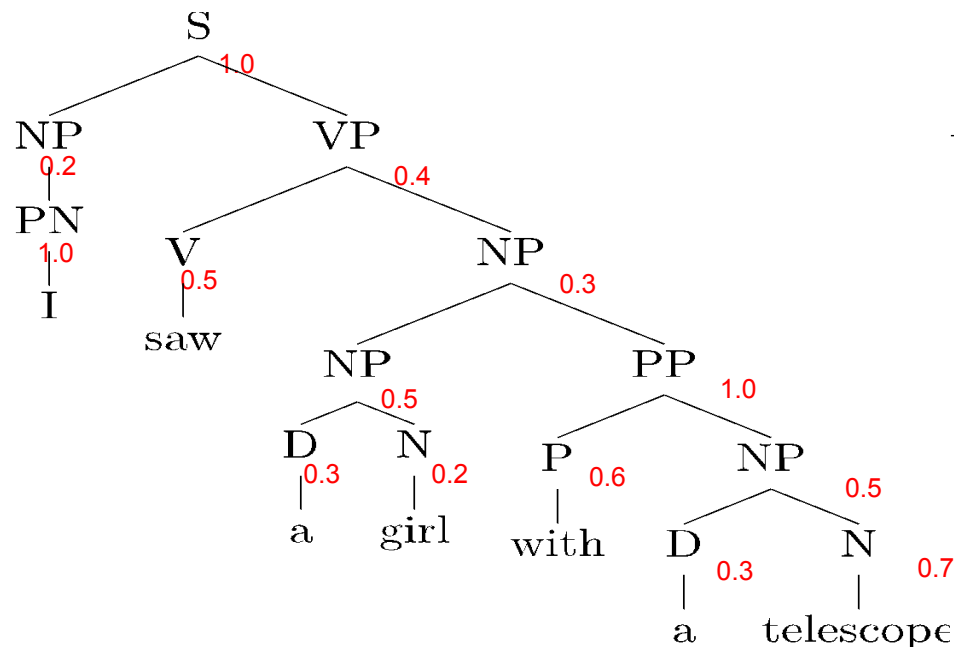
$P \rightarrow with$ 0.6

$P \rightarrow in$ 0.4

$D \rightarrow a$ 0.3

$PP \rightarrow P NP$ 1.0

$D \rightarrow the$ 0.7



$$\begin{aligned}
 p(T) &= 1.0 \times 0.2 \times 1.0 \times 0.4 \times 0.5 \times 0.3 \times \\
 &\quad 0.5 \times 0.3 \times 0.2 \times 1.0 \times 0.6 \times 0.5 \times 0.3 \times 0.7 \\
 &= 2.26 \times 10^{-5}
 \end{aligned}$$



CKY with PCFGs

- Chart is represented by a 3d array of **floats**

`chart [min] [max] [label]`

- It stores probabilities for the most probable subtree with a given signature
- `chart [0] [n] [S]` will store the probability of the most probable full parse tree
-



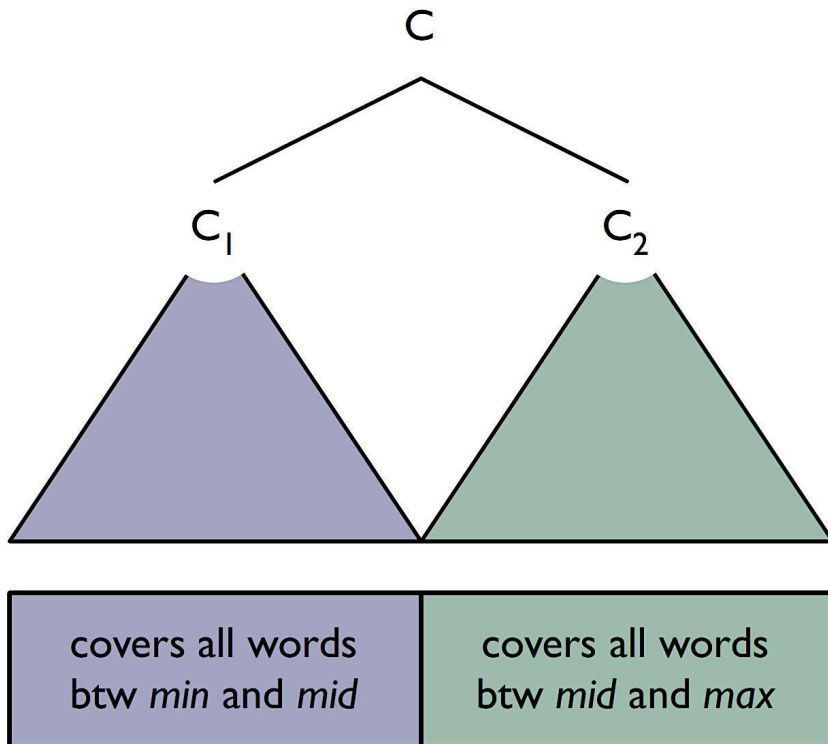
Intuition

$$C \rightarrow C_1 C_2$$

For every C choose C_1, C_2 and mid such that

$$P(T_1) \times P(T_2) \times P(C \rightarrow C_1 C_2)$$

is maximal, where T_1 and T_2 are left and right subtrees.





Implementation: preterminal rules

for each w_i from left to right

for each preterminal rule $C \rightarrow w_i$

$\text{chart}[i - 1][i][C] = p(C \rightarrow w_i)$



Implementation: binary rules

```
for each max from 2 to n
```

```
  for each min from max - 2 down to 0
```

```
    for each syntactic category C
```

```
      double best = undefined
```

```
        for each binary rule  $C \rightarrow C_1 C_2$ 
```

```
          for each mid from min + 1 to max - 1
```

```
            double  $t_1$  = chart[min][mid][ $C_1$ ]
```

```
            double  $t_2$  = chart[mid][max][ $C_2$ ]
```

```
            double candidate =  $t_1 * t_2 * p(C \rightarrow C_1 C_2)$ 
```

```
              if candidate > best then
```

```
                best = candidate
```

```
          chart[min][max][C] = best
```




Unary rules

- Similarly to CFGs: after producing scores for signatures (c, i, j) , try to improve the scores by applying unary rules (and rule chains)
 - If improved, update the scores



Unary (reflexive transitive) closure

$A \rightarrow B$	0.1	\Rightarrow	$A \rightarrow B$	0.1	$A \rightarrow A$	1
$B \rightarrow C$	0.2		$B \rightarrow C$	0.2	$B \rightarrow B$	1
...			$A \rightarrow C$	0.2×0.1	$C \rightarrow C$	1
				...		

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent



Unary (reflexive transitive) closure

The fact that the rule is composite needs to be stored to recover the true tree

$A \rightarrow B$	0.1	\Rightarrow	$A \rightarrow B$	0.1	$A \rightarrow A$	1
$B \rightarrow C$	0.2		$B \rightarrow C$	0.2	$B \rightarrow B$	1
...			$A \rightarrow C$	0.2×0.1	$C \rightarrow C$	1
				...		

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent



Unary (reflexive transitive) closure

The fact that the rule is composite needs to be stored to recover the true tree

$A \rightarrow B$	0.1	\Rightarrow	$A \rightarrow B$	0.1	$A \rightarrow A$	1
$B \rightarrow C$	0.2		$B \rightarrow C$	0.2	$B \rightarrow B$	1
...			$A \rightarrow C$	0.2×0.1	$C \rightarrow C$	1
			

Note that this is not a PCFG anymore as the rules do not sum to 1 for each parent

$A \rightarrow B$	0.1	\Rightarrow	$A \rightarrow B$	0.1	$A \rightarrow A$	1
$B \rightarrow C$	0.2		$B \rightarrow C$	0.1	$B \rightarrow B$	1
$A \rightarrow C$	$1.e - 5$		$A \rightarrow C$	0.02	$C \rightarrow C$	1

What about loops, like: $A \rightarrow B \rightarrow A \rightarrow C$?



Recovery of the tree

- For each signature we store backpointers to the elements from which it was built (e.g., rule and, for binary rules, midpoint)
 - start recovering from $[0, n, S]$
- Be careful with unary rules
 - Basically you can assume that you always used an unary rule from the closure (but it could be the trivial one $C \rightarrow C$)



Speeding up the algorithm (approximate search)

Any ideas?



Speeding up the algorithm

- **Basic pruning (roughly):**
 - For every span (i,j) store only labels which have the probability at most N times smaller than the probability of the most probable label for this span
 - Check not all rules but only rules yielding subtree labels having non-zero probability
- **Coarse-to-fine pruning**
 - Parse with a smaller (simpler) grammar, and precompute (posterior) probabilities for each spans, and use only the ones with non-negligible probability from the previous grammar



Parsing evaluation

- **Intrinsic evaluation:**
 - **Automatic:** evaluate against annotation provided by human experts (gold standard) according to some predefined measure
 - **Manual:** ... according to human judgment
- **Extrinsic evaluation:** score syntactic representation by comparing how well a system using this representation performs on some task
 - E.g., use syntactic representation as input for a semantic analyzer and compare results of the analyzer using syntax predicted by different parsers.



Standard evaluation setting in parsing

- Automatic intrinsic evaluation is used: parsers are evaluated against gold standard by provided by linguists
 - There is a standard split into the parts:
 - training set: used for estimation of model parameters
 - development set: used for tuning the model (initial experiments)
 - test set: final experiments to compare against previous work



Automatic evaluation of constituent parsers

- **Exact match:** percentage of trees predicted correctly
- **Bracket score:** scores how well individual phrases (and their boundaries) are identified
- **Crossing brackets:** percentage of phrases boundaries crossing

The most standard measure;
we will focus on it

A blue callout box with a pointer pointing to the 'Crossing brackets' item in the list above.



Brackets scores

Subtree signatures for
CKY

- The most standard score is **bracket score**
- It regards a tree as a collection of brackets: $[min, max, C]$
- The set of brackets predicted by a parser is compared against the set of brackets in the tree annotated by a linguist
- **Precision, recall** and **F1** are used as scores



Preview: F1 bracket score

